جامعة بنها

كلية الحاسبات والذكاء الإصطناعي

Calculus

Lecture 02

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Chapter 1 Topics

- Numbers and Sets.
- Representations of Functions.
- Domain & Range of Functions.
- Algebra of Functions.
- Increasing and Decreasing.
- Test for Even and Odd Functions.
- Types of Functions and their Graphs.
- Transformations of Functions.

Example9:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x - 2}}$$

Domain & Range (33/37)

Example9:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x - 2}}$$

Numerator is defined for all real numbers.

Denominator is defined for $x - 2 > 0$. Then, $x > 2$.

So, the domain of the function is the interval $(2, \infty)$.

Domain & Range (34/37)

Example10:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x} - 2}$$

Domain & Range (34/37)

Example10:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x} - 2}$$

Numerator is defined for all real numbers.

Denominator is defined for:

1) $x \geq 0$ and 2) $\sqrt{x} - 2 \neq 0$ (i.e., $x \neq 4$)

So, the domain of the function is the interval $[0, \infty) - \{4\}$.

Domain & Range (35/37)

Example11:

Find the domain and range of the following function.

$$f(x) = \frac{3}{x}$$

Domain & Range (35/37)

Example11:

Find the domain and range of the following function.

$$f(x) = \frac{3}{x}$$

The domain of the function is $\mathbb{R} - \{0\}$.

For the range, $y = \frac{3}{x}$, then $x = \frac{3}{y}$

So, the range of the function is $\mathbb{R} - \{0\}$.

Domain & Range (36/37)

Example12:

Find the domain and range of the following function.

$$f(x) = \frac{3x - 4}{x}$$

Example12:

Find the domain and range of the following function.

$$f(x) = \frac{3x - 4}{x}$$

The domain of the function is $\mathbb{R} - \{0\}$.

For the range, $y = \frac{3x-4}{x}$, then $x = \frac{-4}{y-3}$

So, the range of the function is $\mathbb{R} - \{3\}$.

Domain & Range (37/37)

Example13:

Find the domain and range of the following function.

$$f(x) = x^2 + 4$$

Example13:

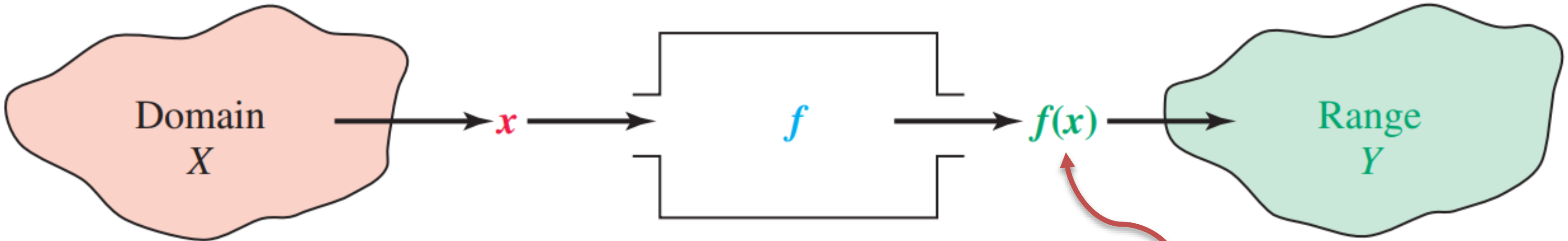
Find the domain and range of the following function.

$$f(x) = x^2 + 4$$

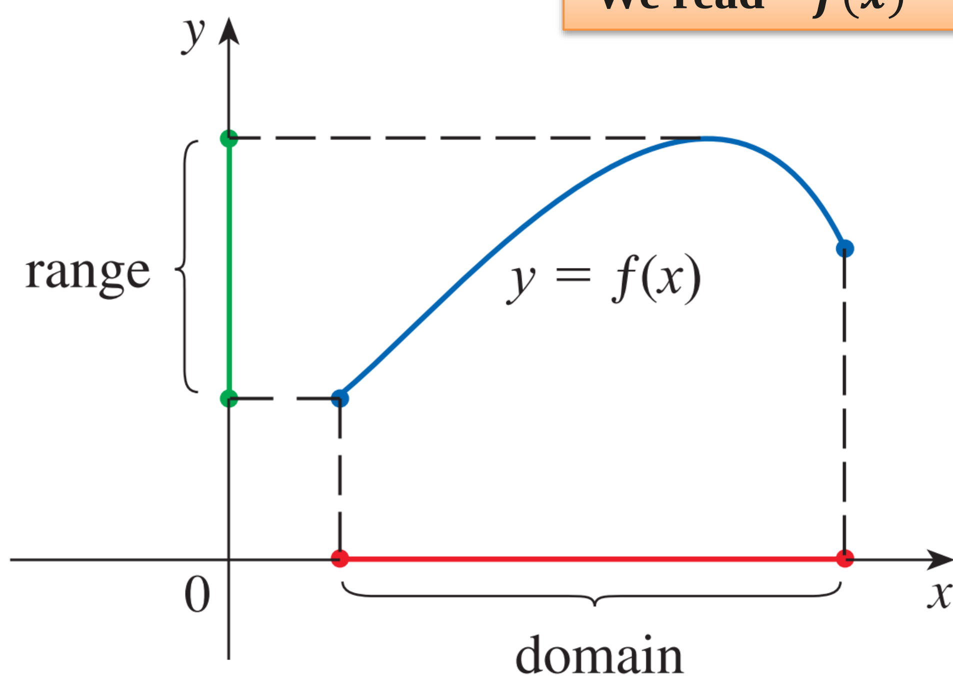
The domain of the function is all the real numbers.

For the range, $y = x^2 + 4$. Therefore, $x^2 = y - 4$, then $x = \sqrt{y - 4}$. Therefore, $y \geq 4$. So, the range of the function is the interval $[4, \infty)$.

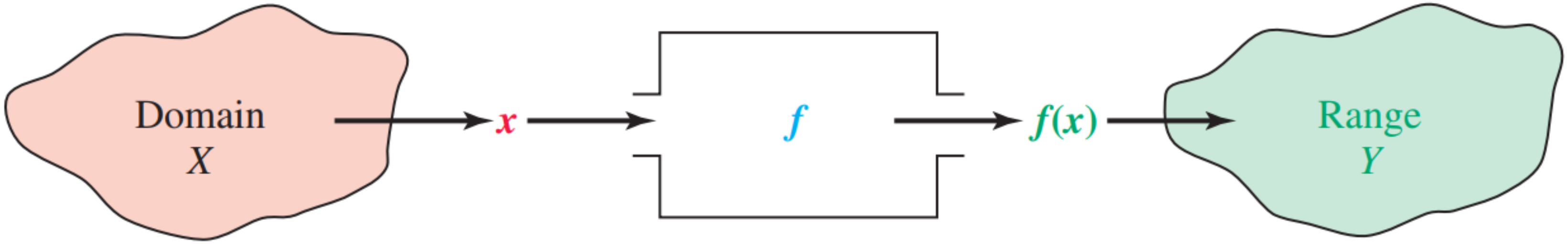
Function (1/4)



We read " *f* (*x*) " as " *f* of *x* "



Function (2/4)



In 1730-1735,

Leonhard Euler used the word "function" to describe any expression made up of a variable and some constants.

He introduced the notation $y = f(x)$.

Leonhard Euler (Switzerland)

Function (3/4)

Example:

$$f(x) = x^2 - x + 4$$

Name: f

Input (independent variable): x

Output (dependent variable): $y = x^2 - x + 4$

Function (3/4)

Example:

$$\boldsymbol{f(x) = x^2 - x + 4}$$

$$f(1) =$$

$$f(0) =$$

$$f(-2) =$$

Function (3/4)

Example:

$$\boldsymbol{f(x) = x^2 - x + 4}$$

$$f(1) = (1)^2 - (1) + 4 = 1 - 1 + 4 = 4$$

$$f(0) = (0)^2 - 0 + 4 = 4$$

$$f(-2) = (-2)^2 - (-2) + 4 = 4 + 2 + 4 = 10$$

Function (4/4)

Example:

$$\boldsymbol{f(x) = x^2 - x + 4}$$

$$f(a) =$$

$$f(2b) =$$

Function (4/4)

Example:

$$\boldsymbol{f(x) = x^2 - x + 4}$$

$$f(a) = (a)^2 - (a) + 4$$

$$f(2b) = (2b)^2 - (2b) + 4$$

Algebra of Functions (1/5)

Definition:

If f and g are functions, then for every x that belongs to the domains of both f and g (that is, for $\boldsymbol{x} \in \boldsymbol{D}(\boldsymbol{f}) \cap \boldsymbol{D}(\boldsymbol{g})$), we define functions $f+g$, $f-g$, fg and f/g by the formulas

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad (\text{ where } g(x) \neq 0)$$

Algebra of Functions (2/5)

Definition:

Functions can also be multiplied by constants: if c is a real number, then the function cf is defined for all x in the domain of f

$$(cf)(x) = cf(x)$$

Algebra of Functions (3/5)

Example :

The functions defined by the formulas

$$f(x) = \sqrt{x} \quad and \quad g(x) = \sqrt{1 - x}$$

Find $D(f) \cap D(g)$

Algebra of Functions (3/5)

Example :

The functions defined by the formulas

$$f(x) = \sqrt{x} \quad and \quad g(x) = \sqrt{1 - x}$$

$D(f) = [0, \infty)$

$D(g) = (-\infty, 1]$

Then, $D(f) \cap D(g) = [0, \infty) \cap (-\infty, 1] = [0, 1]$

Algebra of Functions (4/5)

Example : $f(x) = \sqrt{x}$ *and* $g(x) = \sqrt{1-x}$

Function	Formula	Domain
$f+g$	$(f+g)(x) = \sqrt{x} + \sqrt{1-x}$	$[0,1] = D(f) \cap D(g)$
$f-g$	$(f-g)(x) = \sqrt{x} - \sqrt{1-x}$	$[0,1]$
$g-f$	$(g-f)(x) = \sqrt{1-x} - \sqrt{x}$	$[0,1]$
$f \cdot g$	$(f \cdot g)(x) = f(x)g(x) = \sqrt{x(1-x)}$	$[0,1]$
f/g	$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x}{1-x}}$	$[0,1)(x = 1 \; excluded)$
g/f	$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \sqrt{\frac{1-x}{x}}$	$(0,1](x = 0 \; excluded)$

Algebra of Functions (5/5)

Example :

The functions defined by the formulas

$$f(x) = \sqrt{9 - x^2} \quad \textit{and} \quad g(x) = \sqrt{x^2 - 1}$$

Find $D(f) \cap D(g)$

Algebra of Functions (5/5)

Example :

The functions defined by the formulas

$$f(x) = \sqrt{9 - x^2} \quad \textit{and} \quad g(x) = \sqrt{x^2 - 1}$$

$D(f) = [-3, 3]$

$D(g) = (-\infty, -1] \cup [1, \infty)$

Then, $D(f) \cap D(g) =$

Algebra of Functions (5/5)

Example :

The functions defined by the formulas

$$f(x) = \sqrt{9 - x^2} \quad \textit{and} \quad g(x) = \sqrt{x^2 - 1}$$

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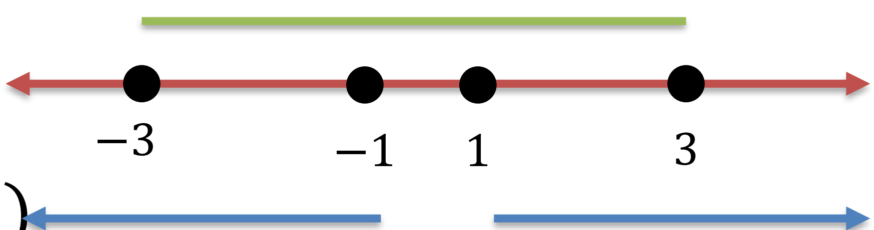


$D(g) = (-\infty, -1] \cup [1, \infty)$

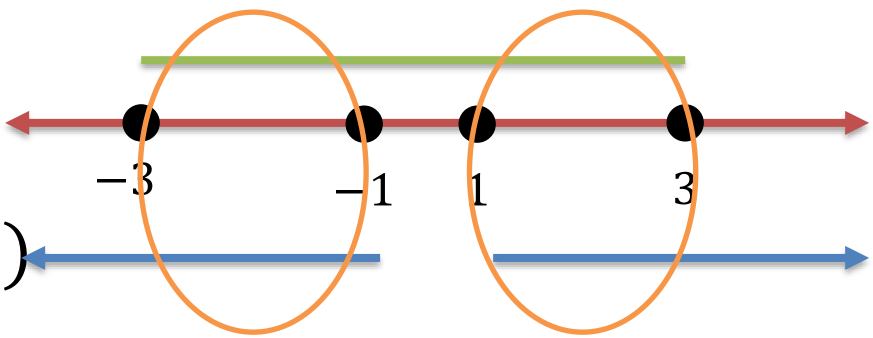
Then, $D(f) \cap D(g) =$

Algebra of Functions (5/5)

Example :

The functions defined by the formulas

$$f(x) = \sqrt{9 - x^2} \quad \textit{and} \quad g(x) = \sqrt{x^2 - 1}$$



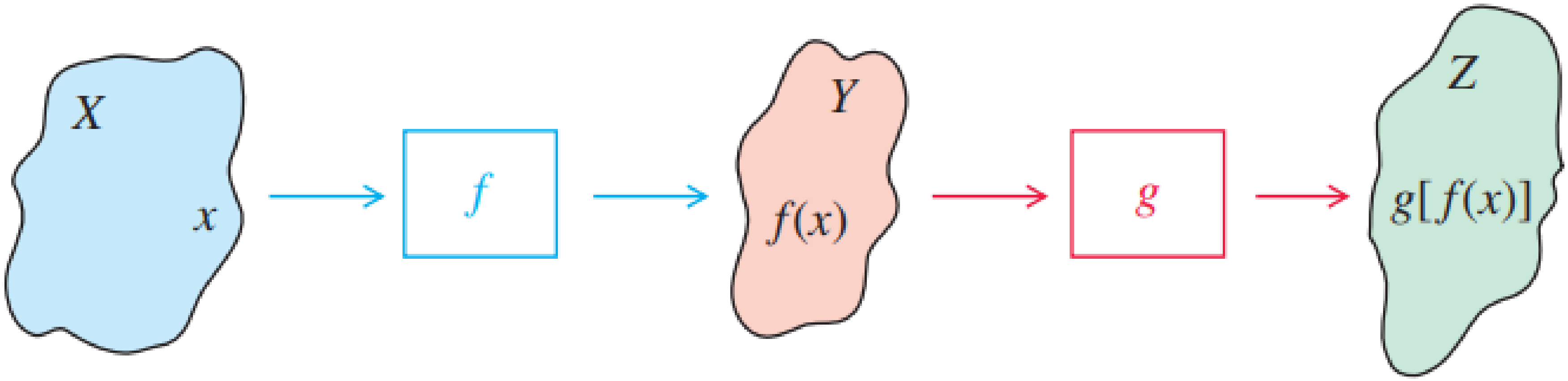
$D(f) = [-3, 3]$

$D(g) = (-\infty, -1] \cup [1, \infty)$

Then, $D(f) \cap D(g) = [-3, -1] \cup [1, 3]$

The Composite Function:

Let f and g be functions. The composite function, or composition, of g and f is the function whose values are given by $g(f(x))$ for all x in the domain of f such that $f(x)$ is in the domain of g. Written as $(g \circ f)(x)$



The Composite Function (2/10)

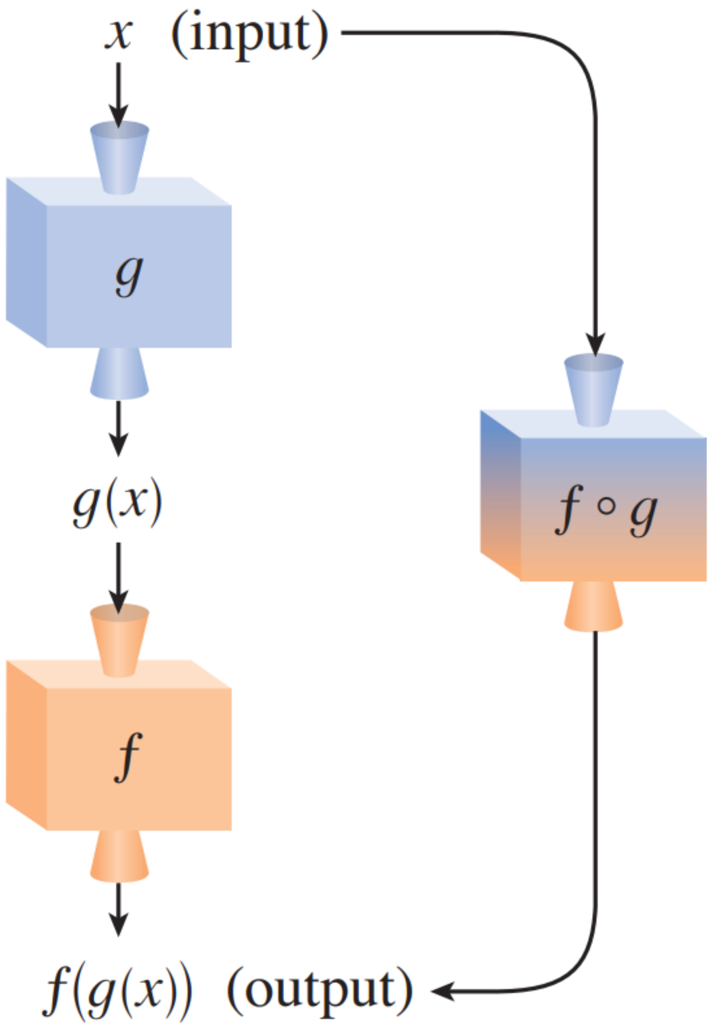
The Composite Function:

The composition of f and g is the function $f \circ g$

("f composed with g") is defined by

$$(f \circ g)(x) = f\big(g(x)\big)$$

To be valid: $R(g) \subseteq D(f)$



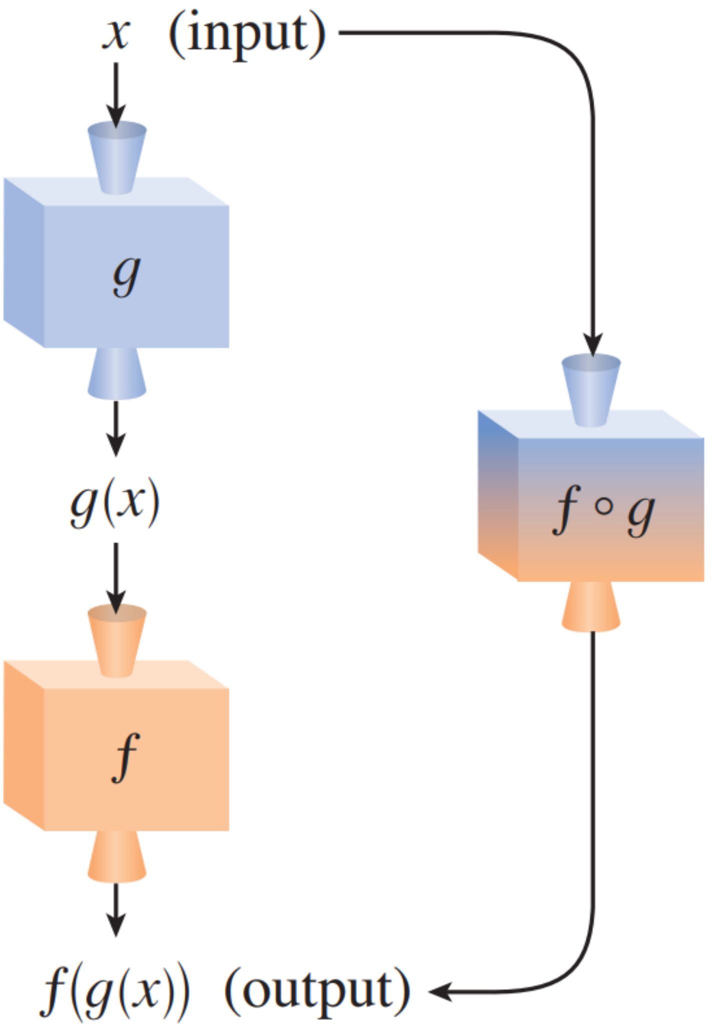
The Composite Function (3/10)

The Composite Function:

$$(f \circ g)(x) = f\big(g(x)\big)$$

The **domain** of $f \circ g$

is the set of values of x in the domain of g such that $g(x)$ lies in the domain of f.



The Composite Function (4/10)

Example:

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $(f \circ f)(x)$

(d) $(g \circ g)(x)$

The Composite Function (4/10)

Example:

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find

$$(a)\ (f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x+1}$$

$$(b)\ (g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$$

$$(c)\ (f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$$

$$(d)\ (g \circ g)(x) = g(g(x)) = g(x) + 1$$

$$= (x+1) + 1 = x + 2$$

The Composite Function (5/10)

Example:

$D(f \circ g)$

$If\ f(x) = \sqrt{x}\ and\ g(x) = x + 1, find$

$(a)\ (f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x+1}$

$\boldsymbol{Domain}\ [-1, \infty)$

The Composite Function (6/10)

$D(f \circ g)$

Example:

$If\ f(x) = \sqrt{x}\ and\ g(x) = x + 1, find$

$(b)\ (g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$

$\textbf{Domain}\ [0, \infty)$

The Composite Function (7/10)

Example:

$D(f \circ g)$

$$If\ f(x) = \sqrt{x}\ and\ g(x) = x + 1, find$$

$$(c)\ (f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$$

$$\boldsymbol{Domain}[0, \infty)$$

The Composite Function (8/10)

Example:

$D(f \circ g)$

$$If\ f(x) = \sqrt{x}\ and\ g(x) = x + 1,\ find$$

$$(d)\ (g \circ g)(x) = g(g(x)) = g(x) + 1$$

$$= (x+1) + 1 = x + 2$$

$$\boldsymbol{Domain}\ (-\infty, \infty).$$

The Composite Function (9/10)

Example:

$D(f \circ g)$

$$if\ f(x) = x^2\ and\ g(x) = \sqrt{x},\ then\ (f \circ g)(x) =$$

The Composite Function (10/10)

Example:

$D(f \circ g)$

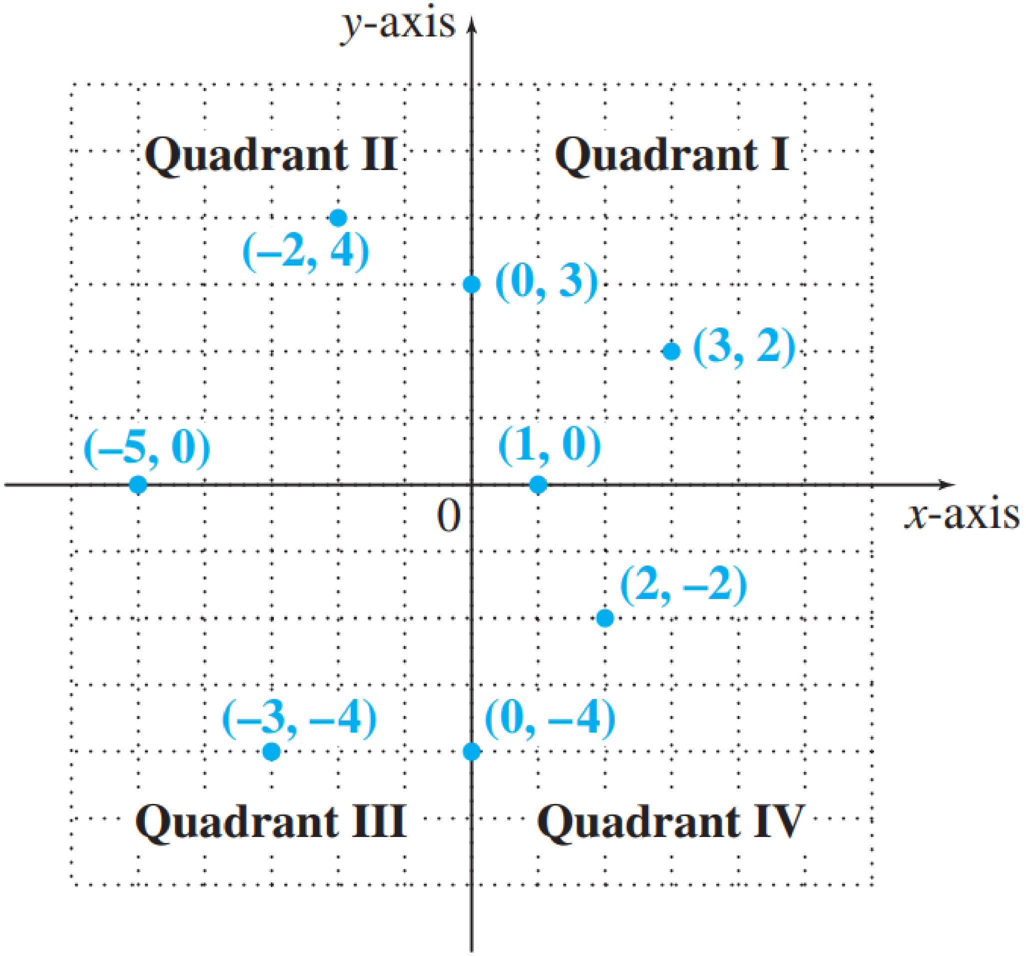
$if\ f(x) = x^2\ and\ g(x) = \sqrt{x},\ then\ (f \circ g)(x) =$

$$(f \circ g)(x) = (\sqrt{x})^2 = x.$$

However, the domain of $f \circ g$ is $[0, \infty)$, not $(-\infty, \infty)$, since $\sqrt{x}$ requires $x \geq 0$.

Graphs of Function (1/3)

Cartesian coordinate:



René Descartes (France)

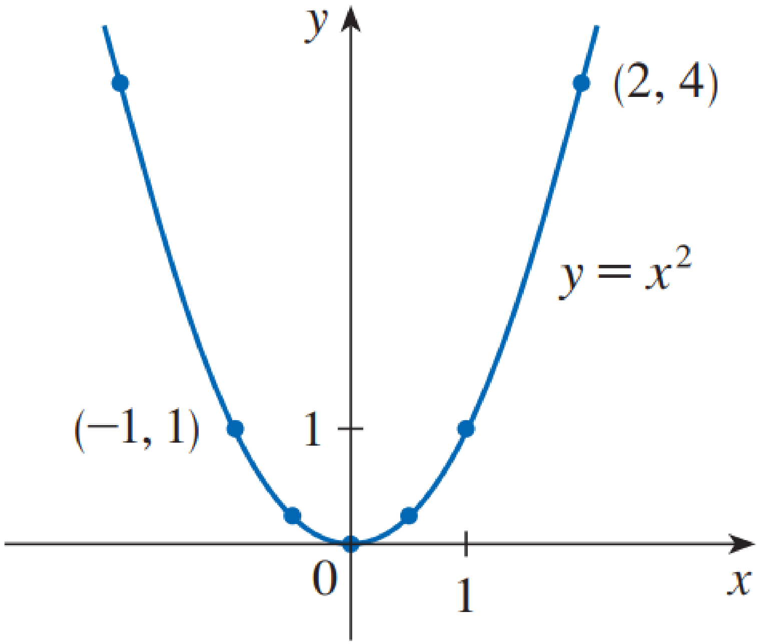
In 1637

Graphs of Function (2/3)

Cartesian coordinate:

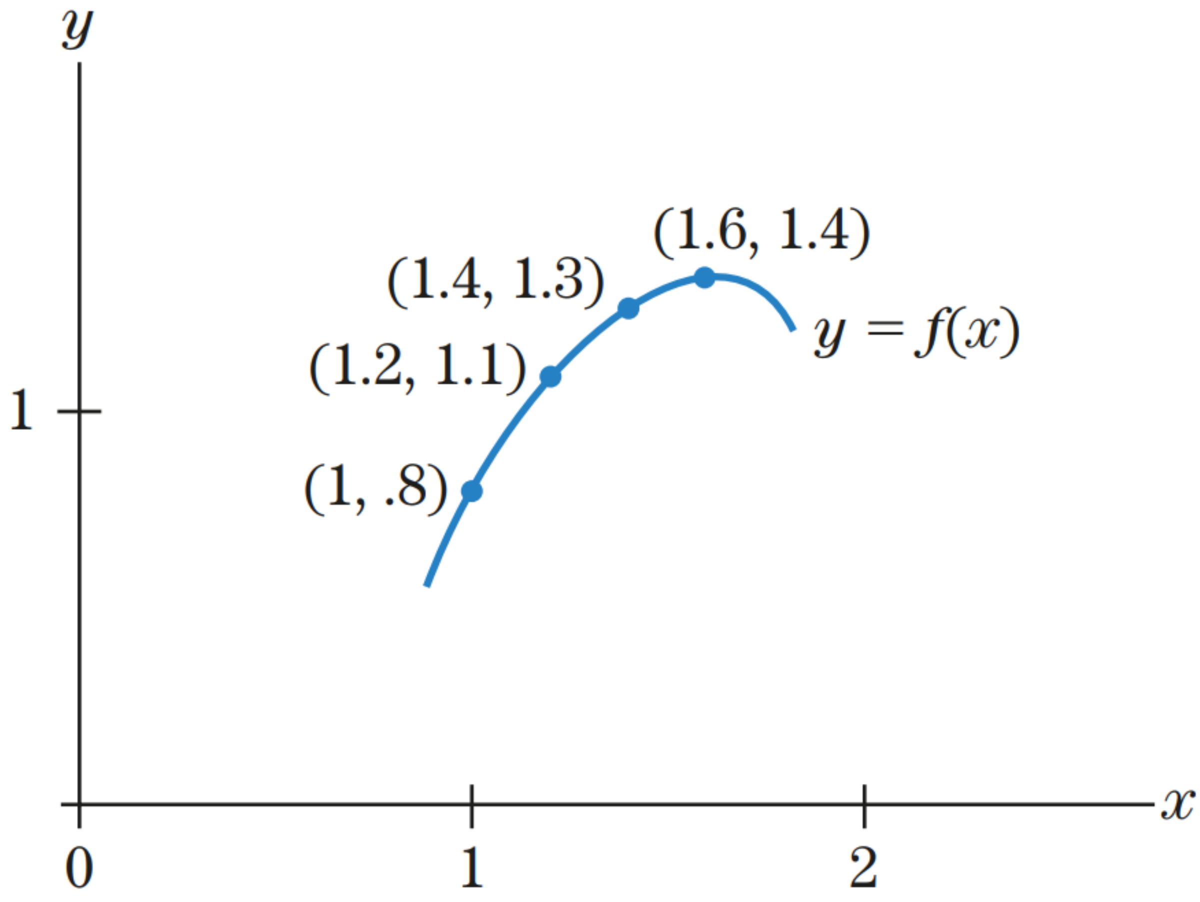
$$f(x) = x^2$$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

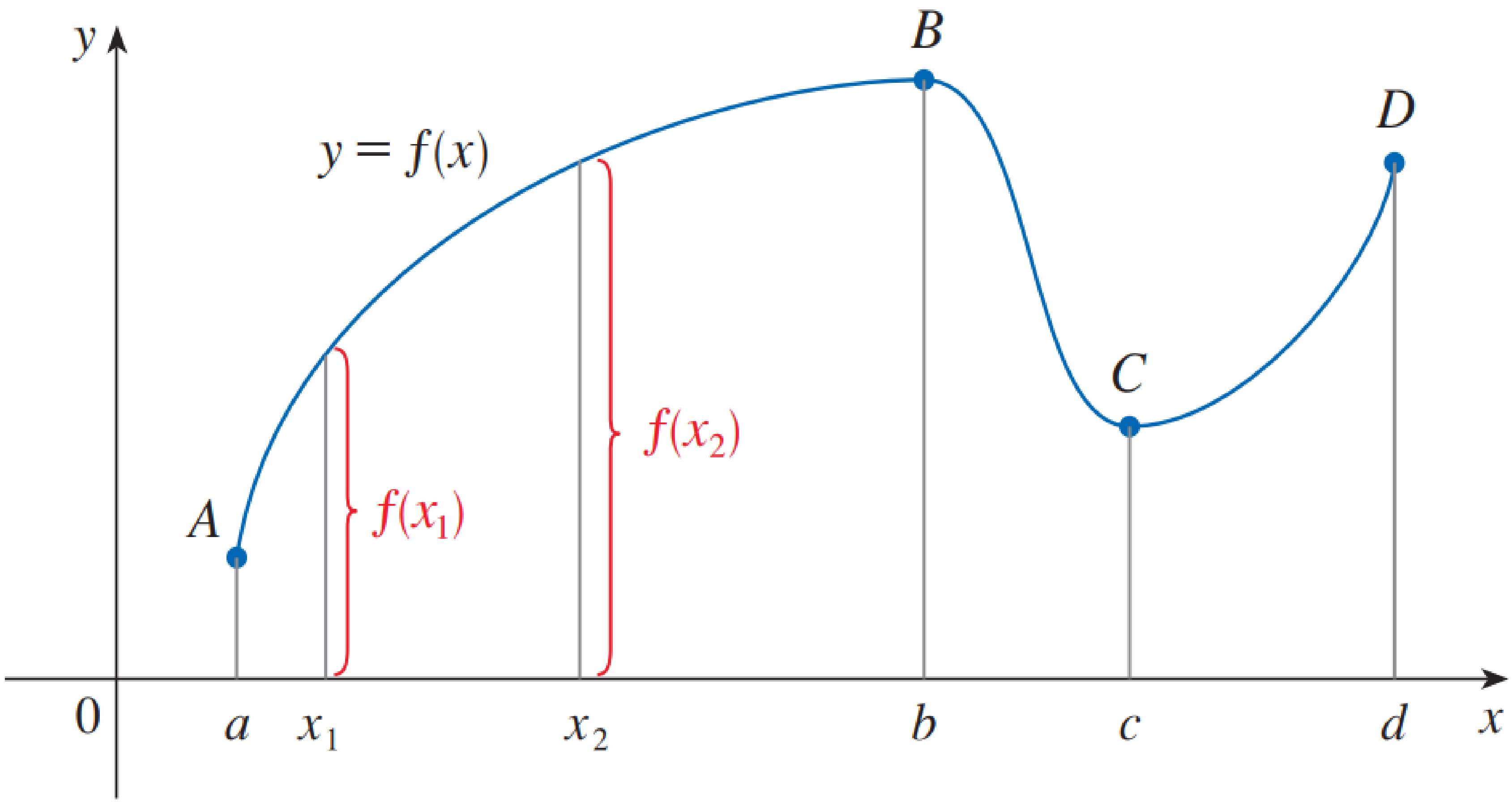


Graphs of Function (3/3)

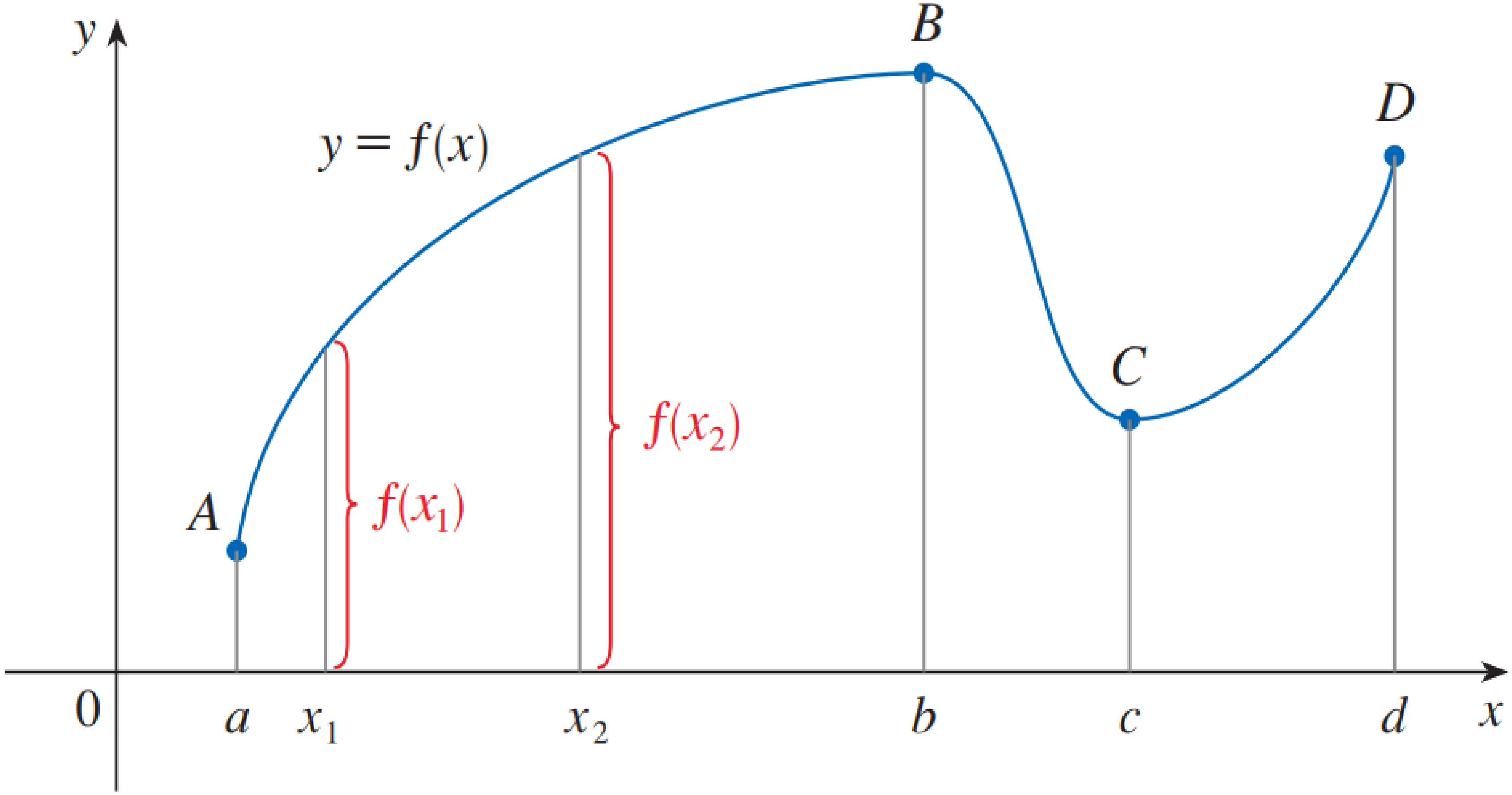
Cartesian coordinate:



Increasing and Decreasing (1/5)



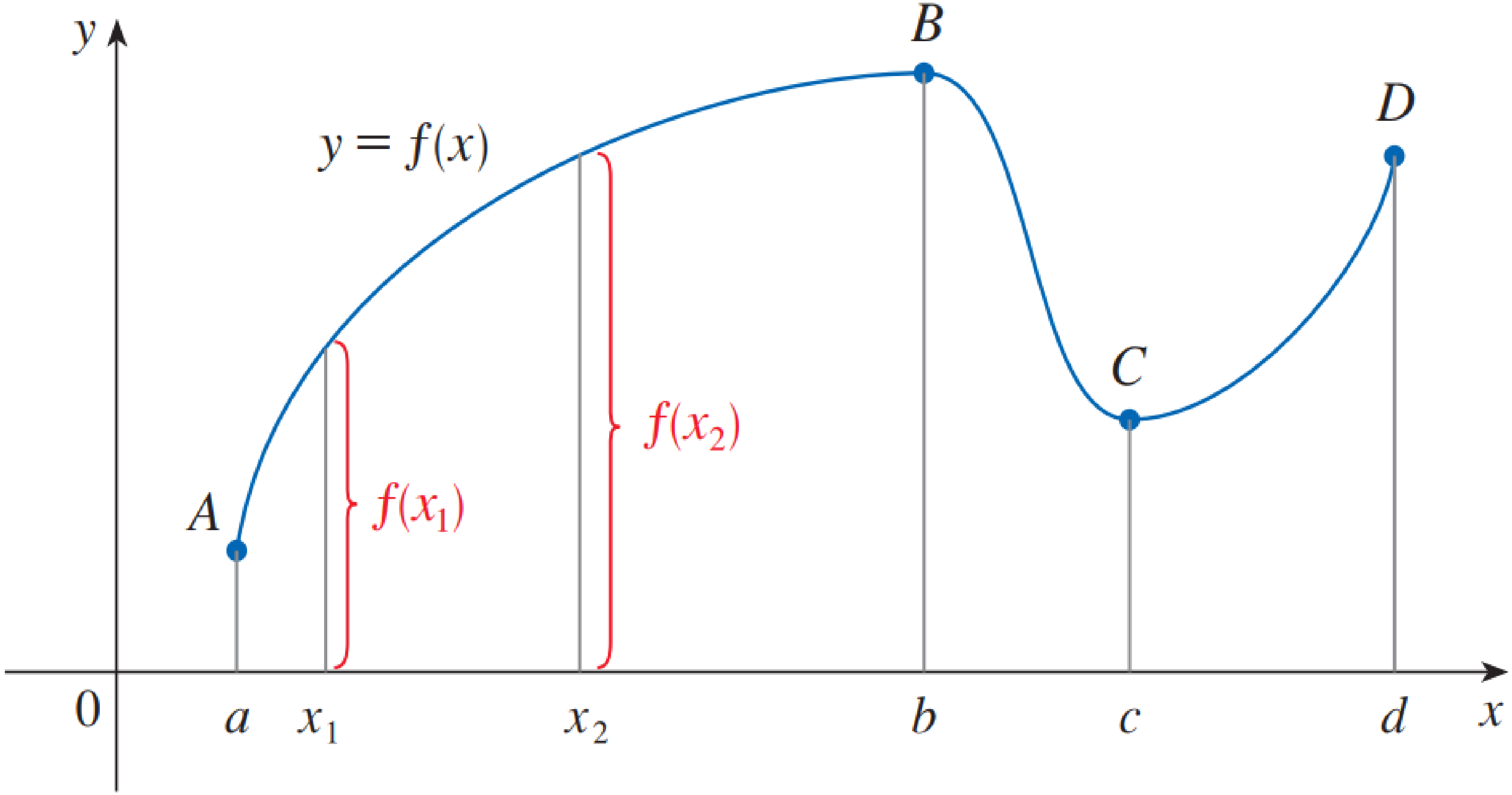
Increasing and Decreasing (2/5)



A function f is called **increasing** on an interval I if

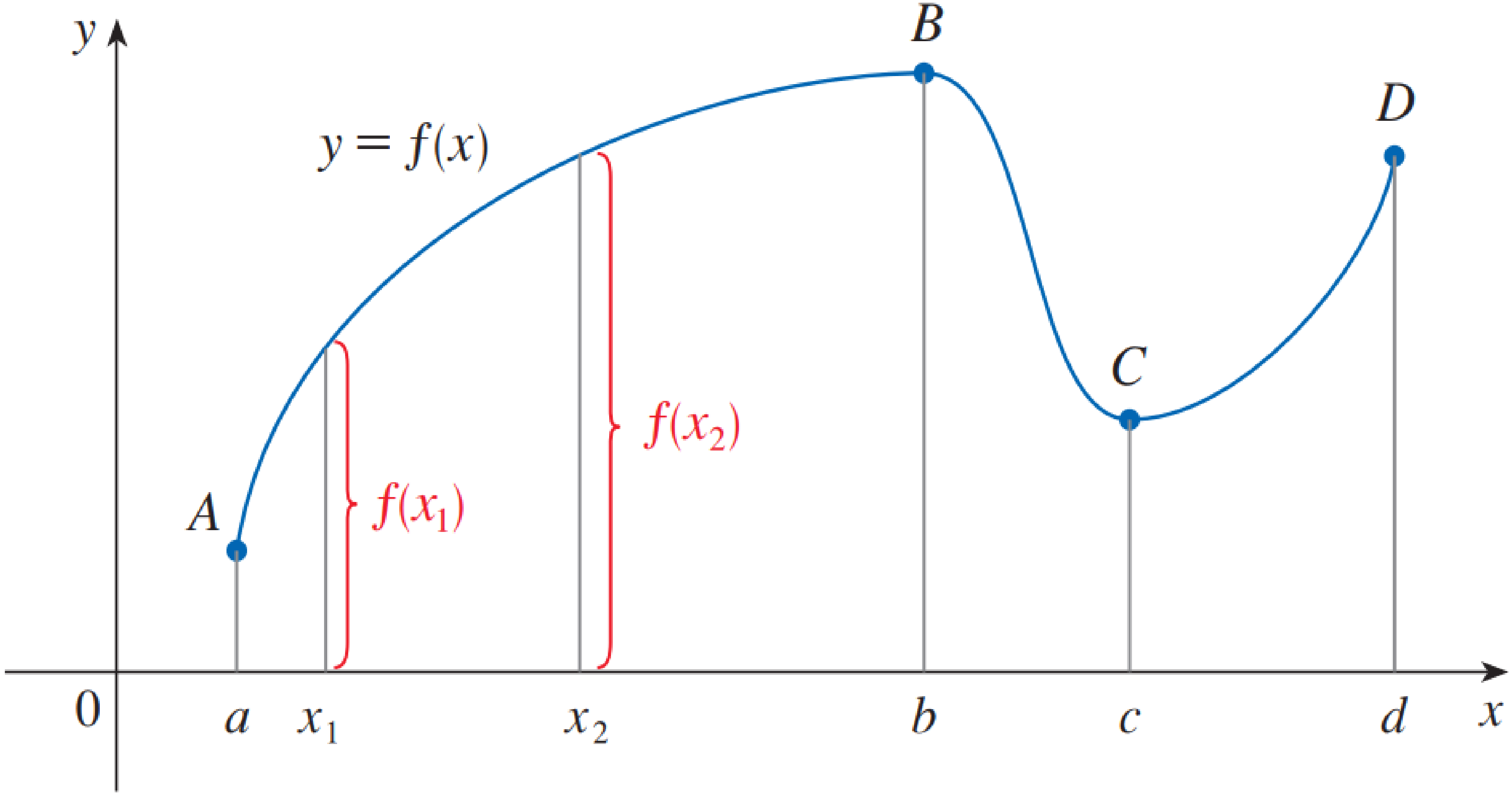
$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

Increasing and Decreasing (2/5)



Therefore, the function *f* is **increasing** on an interval $[a, b]$

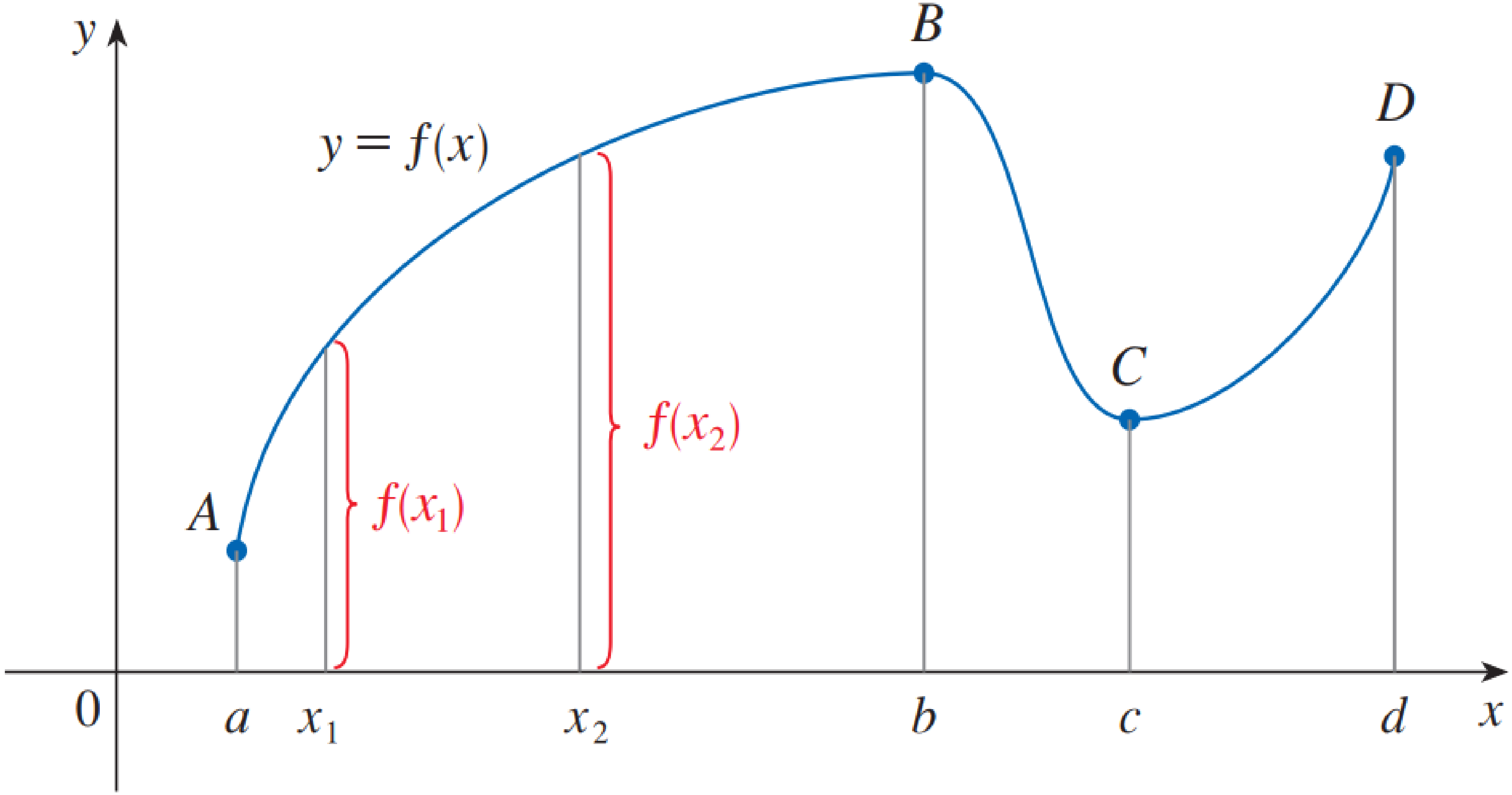
Increasing and Decreasing (3/5)



A function *f* is called **decreasing** on an interval *I* if

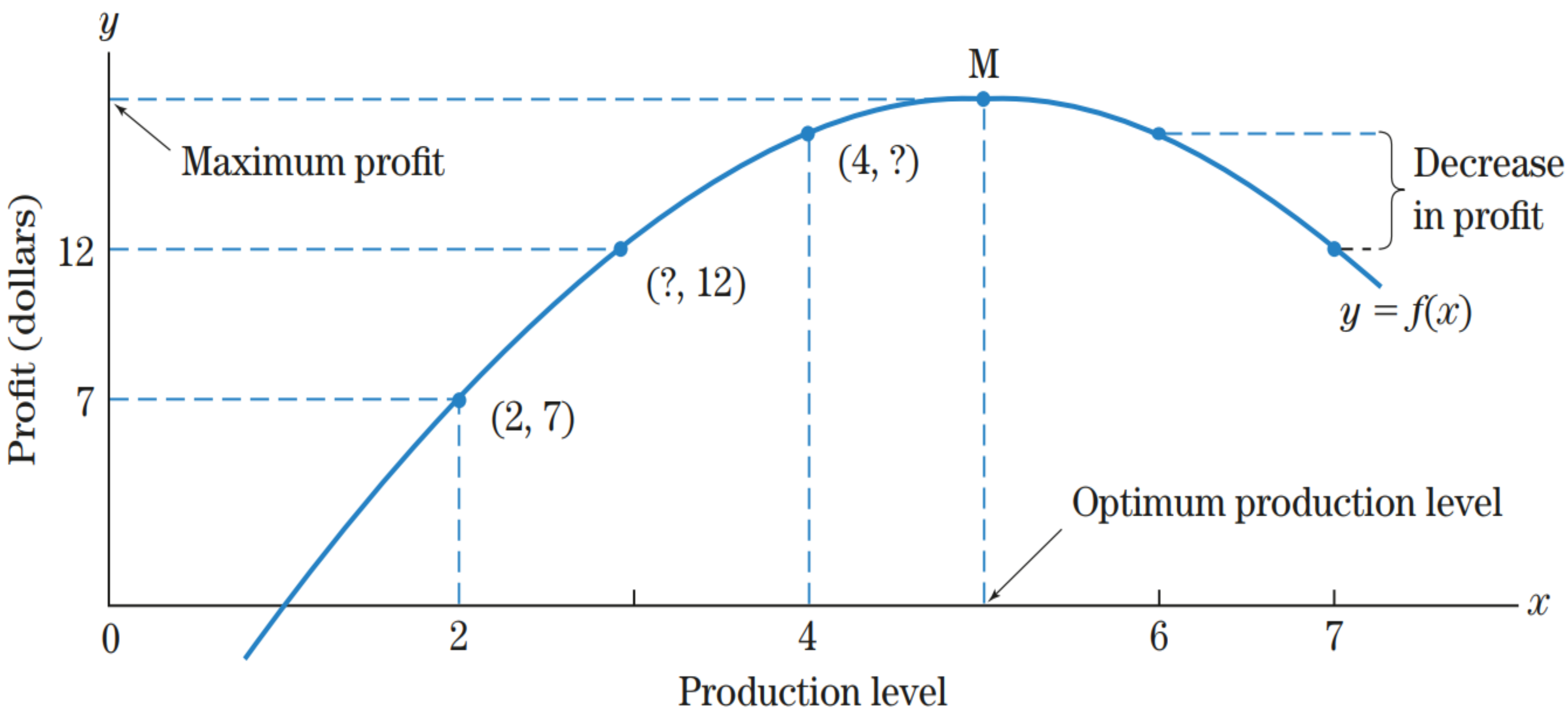
$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

Increasing and Decreasing (3/5)



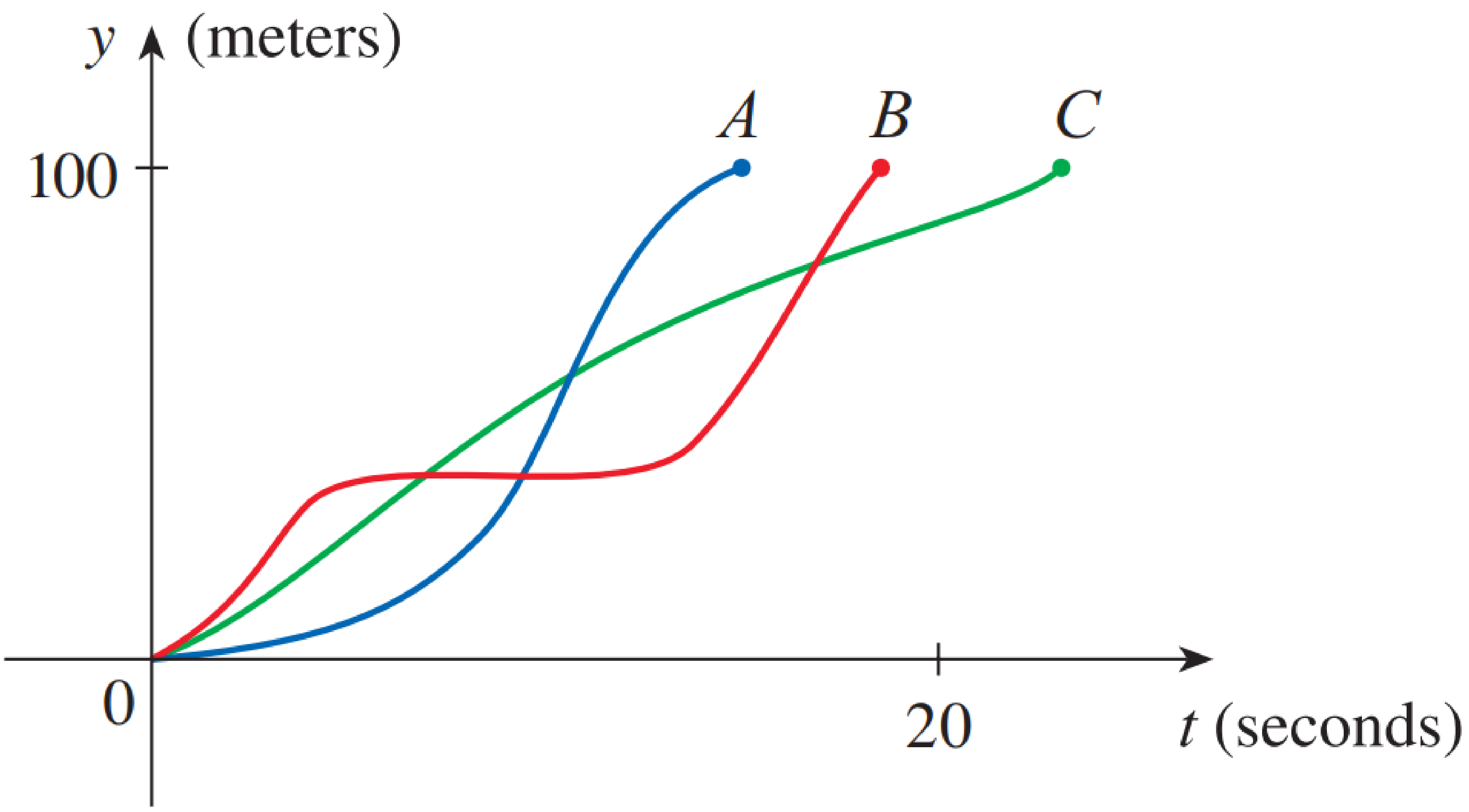
Therefore, the function f is **decreasing** on an interval $[b, c]$

Increasing and Decreasing (4/5)



Increasing and Decreasing (5/5)

100-meter race



Three runners A, B , and C.

Even and Odd Functions (1/10)

Test for Even and Odd Functions

The function $y = f(x)$ is **even** when

$$f(-x) = f(x).$$

The function $y = f(x)$ is **odd** when

$$f(-x) = -f(x).$$

Even and Odd Functions (2/10)

Example:

Determine whether each function is even, odd, or neither.

a. $f(x) = x^3 - x$

b. $g(x) = \dfrac{1}{x^2}$

c. $h(x) = -x^2 - x - 1$

Even and Odd Functions (3/10)

Example:

Determine whether each function is even, odd, or neither.

a. $f(x) = x^3 - x$

This function is odd because

$$f(-x) = (-x)^3 - (-x) = -x^3 + x$$

$$= -(x^3 - x) = -f(x).$$

Even and Odd Functions (4/10)

Example:

Determine whether each function is even, odd, or neither.

b. $g(x) = \dfrac{1}{x^2}$

This function is even because

$$g(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = g(x).$$

Even and Odd Functions (5/10)

Example:

Determine whether each function is even, odd, or neither.

c. $h(x) = -x^2 - x - 1$ **(1/2)**

Substituting $-x$ for x produces

$$h(-x) = -(-x)^2 - (-x) - 1 = -x^2 + x - 1.$$

Because $h(x) = -x^2 - x - 1$ and $-h(x) = x^2 + x + 1$,

$h(-x) \neq h(x)$ Function is *not* even.

Even and Odd Functions (5/10)

Example:

Determine whether each function is even, odd, or neither.

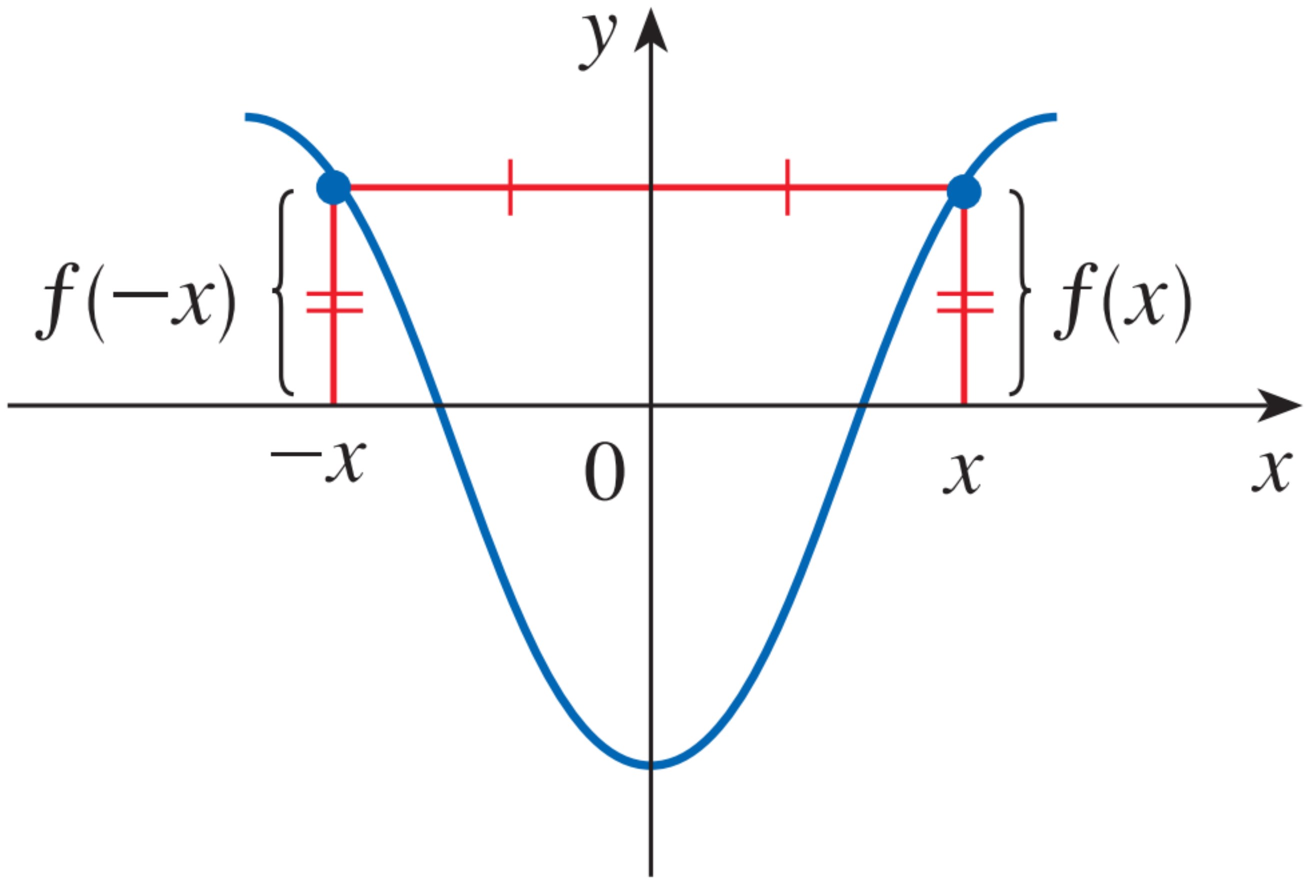
c. $h(x) = -x^2 - x - 1$ **(2/2)**

and

$$h(-x) \neq -h(x).$$ Function is *not* odd.

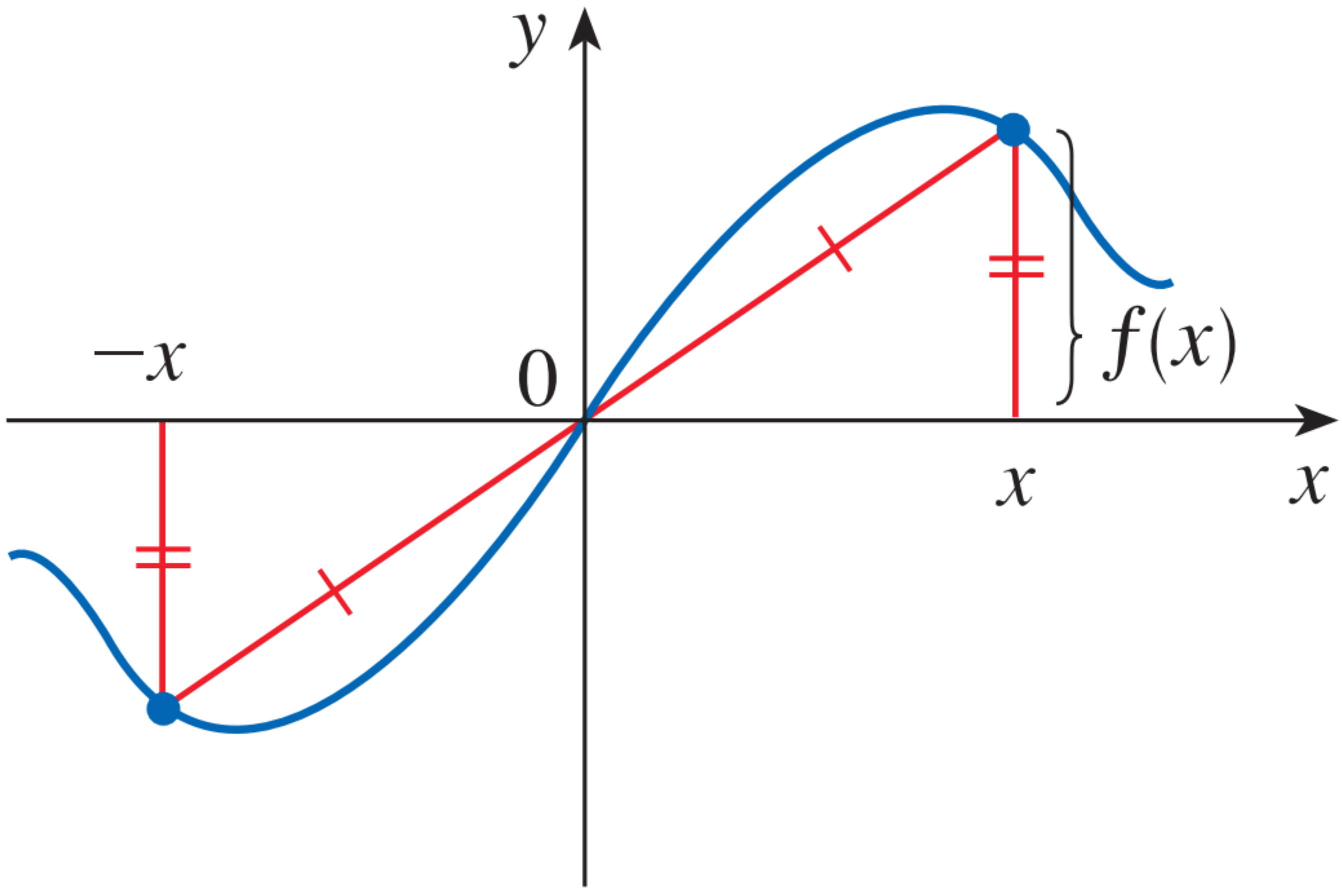
So, the function is neither even nor odd.

Even and Odd Functions (6/10)



An even function

Even and Odd Functions (7/10)

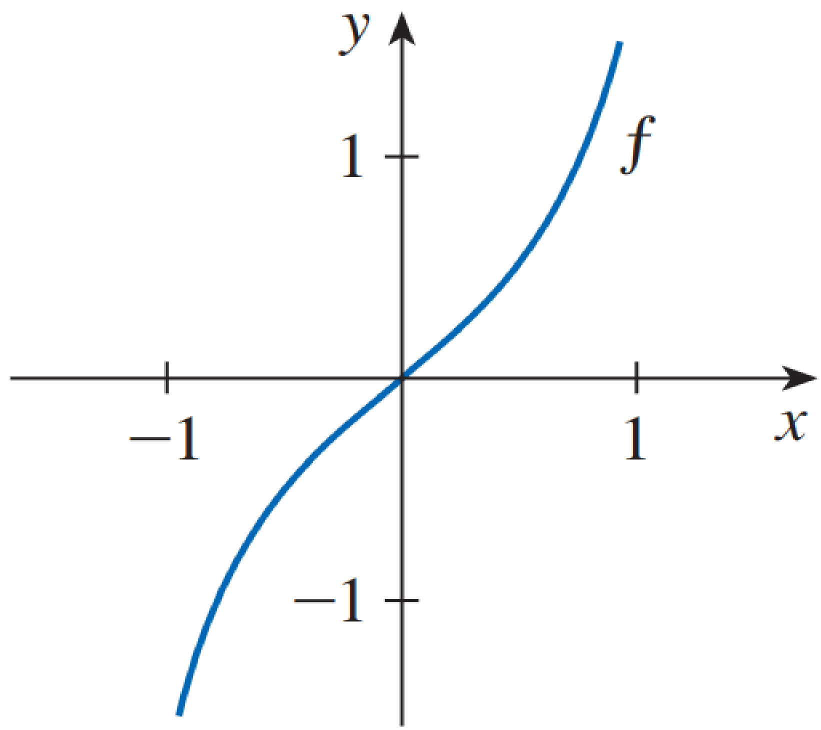


An odd function

Even and Odd Functions (8/10)

Example:

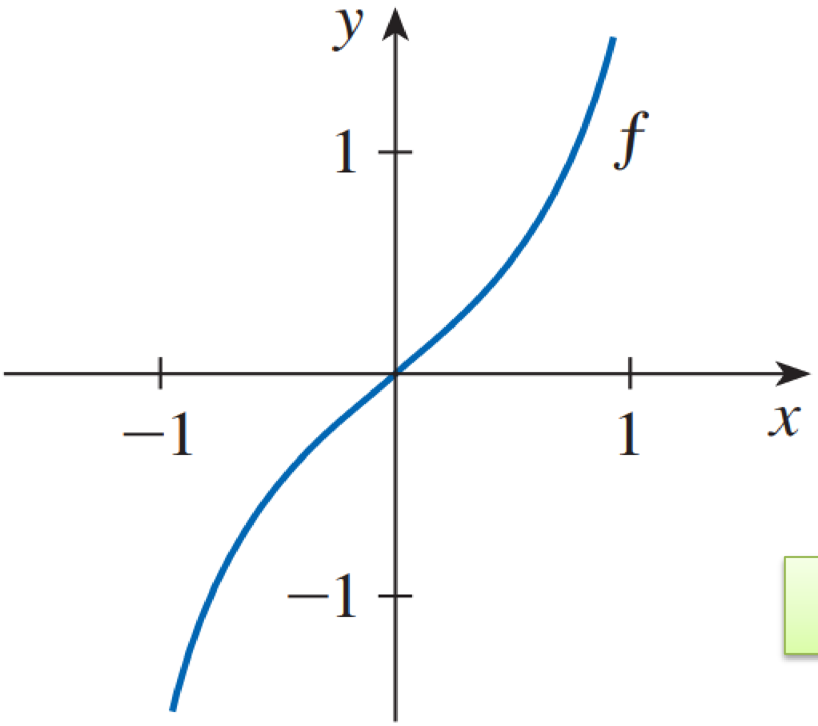
Determine whether each function is even, odd, or neither.



Even and Odd Functions (8/10)

Example:

Determine whether each function is even, odd, or neither.

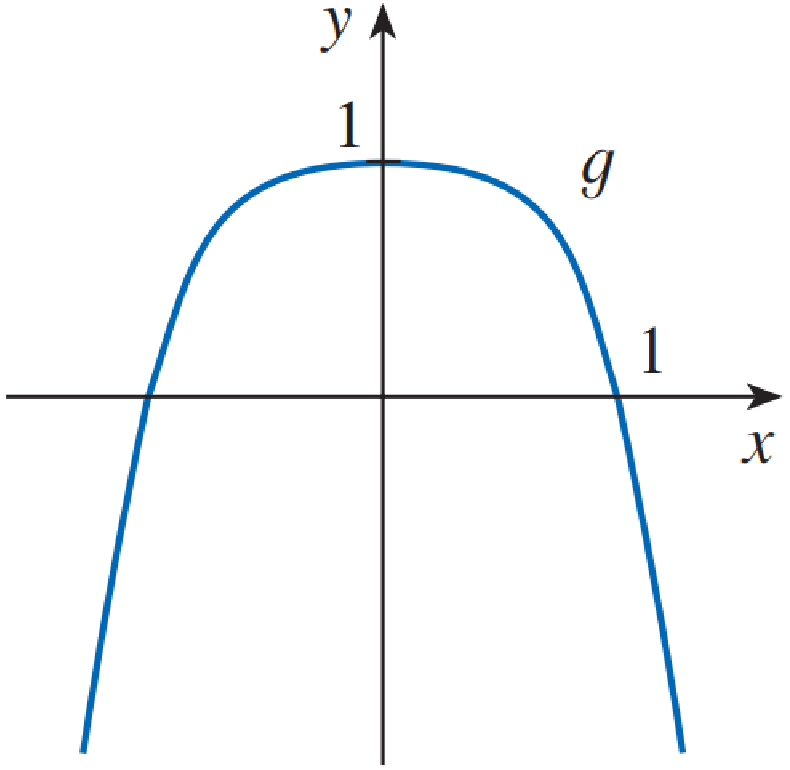


Odd

Even and Odd Functions (9/10)

Example:

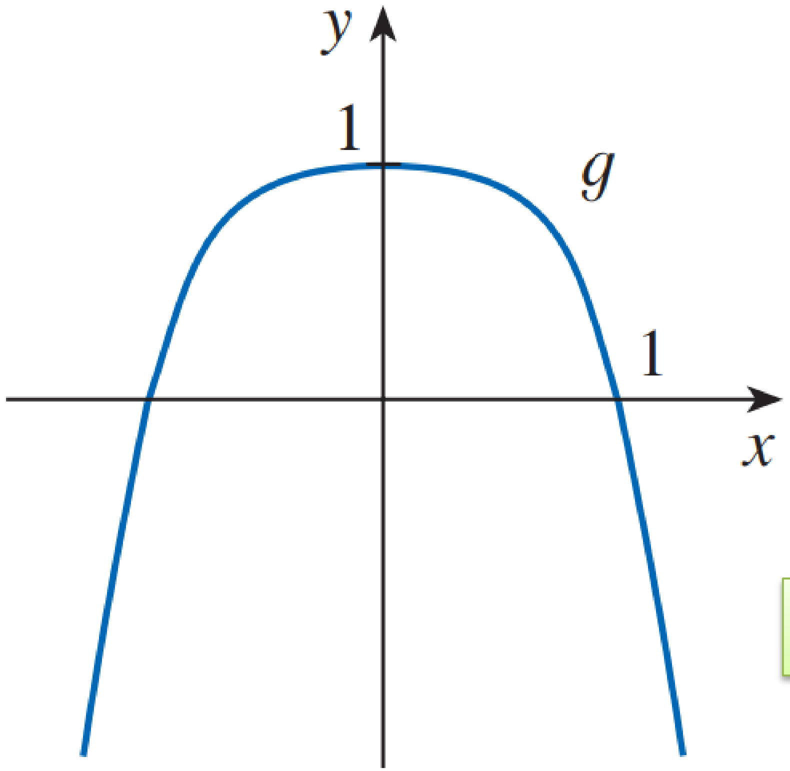
Determine whether each function is even, odd, or neither.



Even and Odd Functions (9/10)

Example:

Determine whether each function is even, odd, or neither.

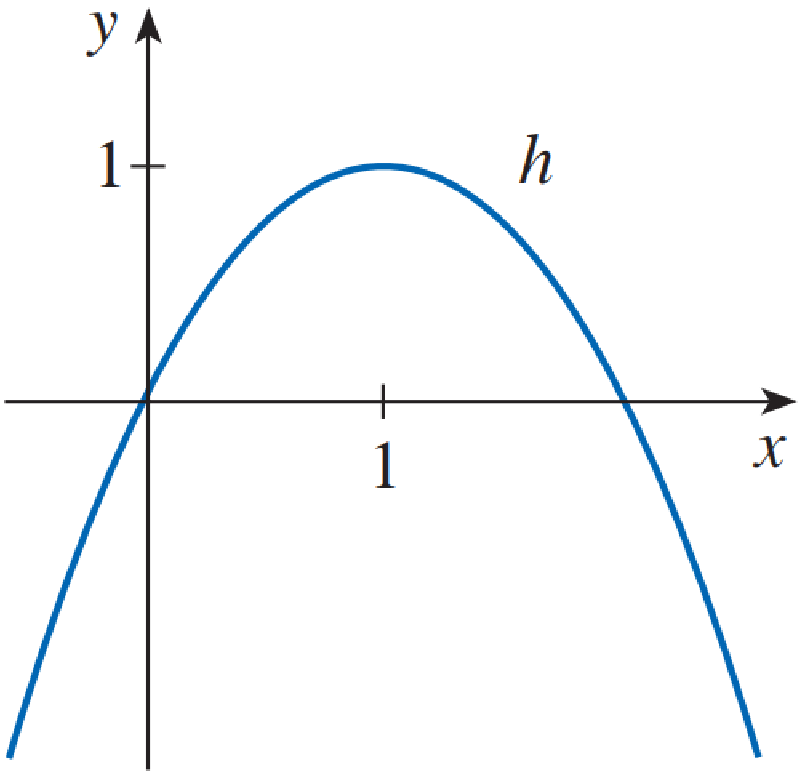


Even

Even and Odd Functions (10/10)

Example:

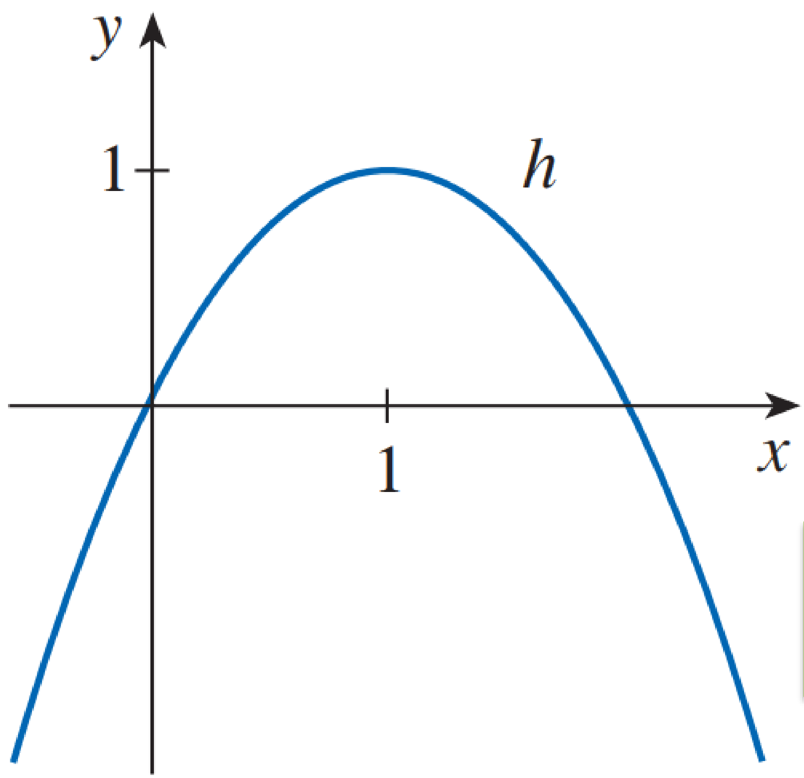
Determine whether each function is even, odd, or neither.



Even and Odd Functions (10/10)

Example:

Determine whether each function is even, odd, or neither.



Neither Even nor Odd

Types of Fun. & Graph (1/18)

Elementary functions fall into three categories:

1. Algebraic functions (Polynomial, Radical, Rational).
2. Trigonometric functions (sine, cosine, tangent, …).
3. Exponential and logarithmic functions.

Polynomial (1/3):

The most common type of algebraic function is a polynomial function: A function P is called a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots,$ an are constants called the coefficients of the polynomial. The domain of any polynomial is $\mathbb{R}$.

Types of Fun. & Graph (2/18)

Polynomial (2/3):

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

If the **leading coefficient** $a_n \neq 0$, then the degree of the polynomial is n. For example, the function

$$P(x) = 2x^6 - x^4 + \frac{2}{5}x^3 + \sqrt{2}$$

is a polynomial of degree 6.

Types of Fun. & Graph (2/18)

Polynomial (3/3):

If $a \neq 0$,

Zeroth degree: $f(x) = a$ Constant function

First degree: $f(x) = ax + b$ Linear function

Second degree: $f(x) = ax^2 + bx + c$ Quadratic function

Third degree: $f(x) = ax^3 + bx^2 + cx + d$ Cubic function

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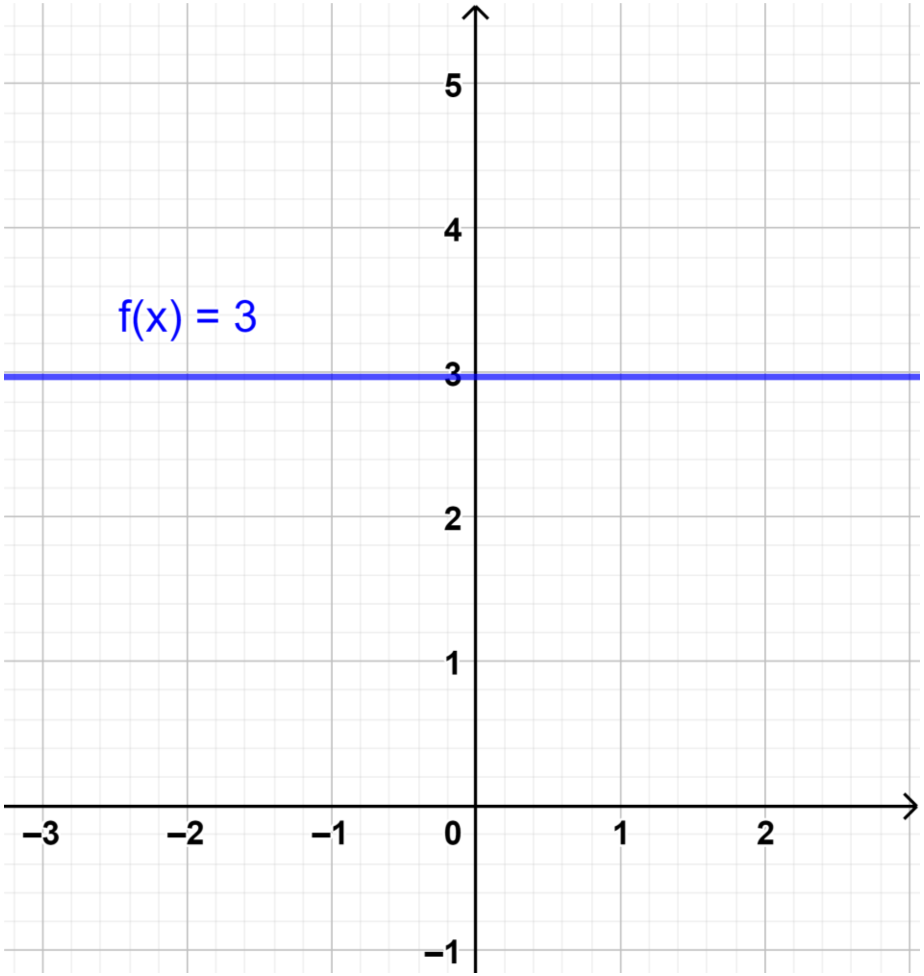
Constant Function (1/3):

Zeroth degree: $f(x) = a$

If $a = 3$



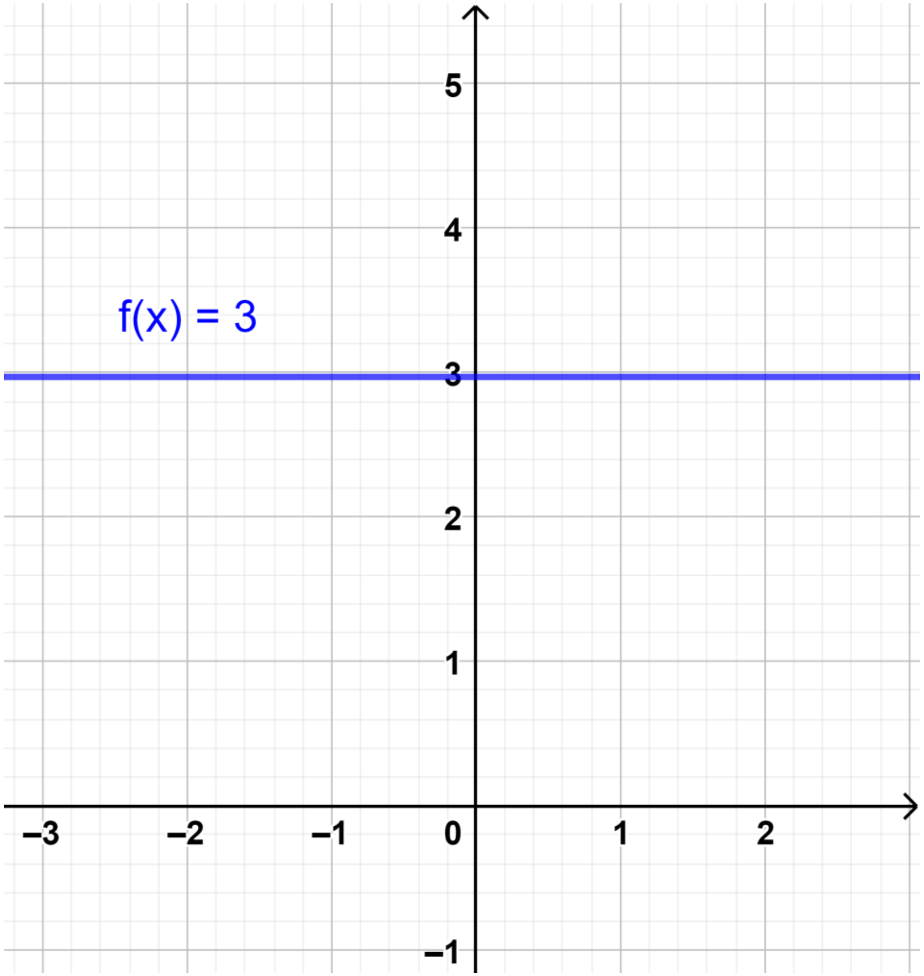
Constant Function (2/3):

Zeroth degree: $f(x) = a$

If $a = 3$

Domain = ??

Range = ??



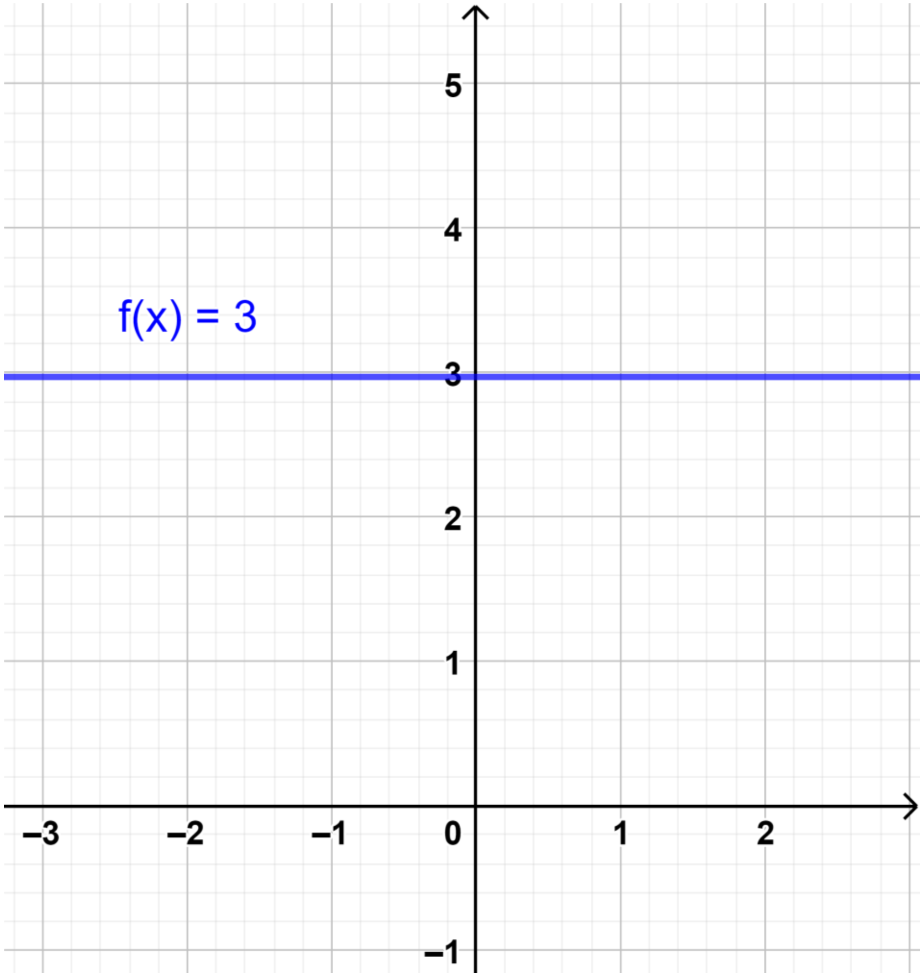
Constant Function (2/3):

Zeroth degree: $f(x) = a$

If $a = 3$

Domain = $\mathbb{R}$

Range = {3}



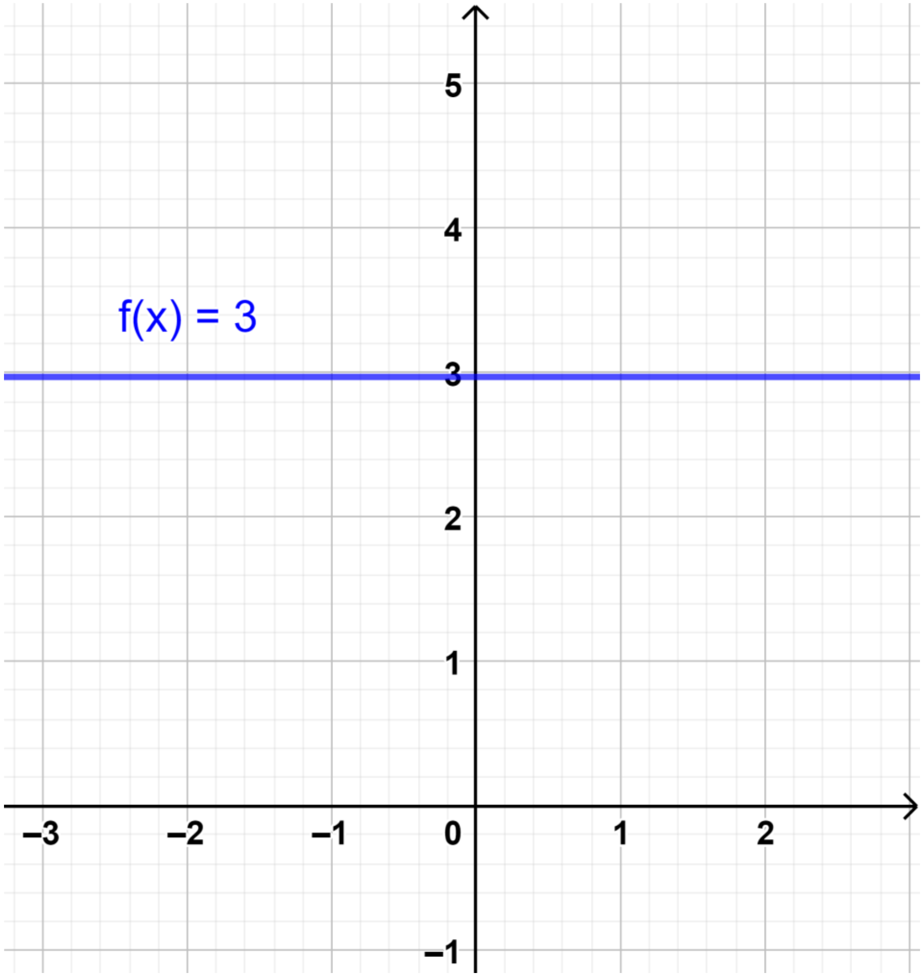
Types of Fun. & Graph (3/18)

Constant Function (3/3):

Zeroth degree: $f(x) = a$

If $a = 3$

Odd or Even??

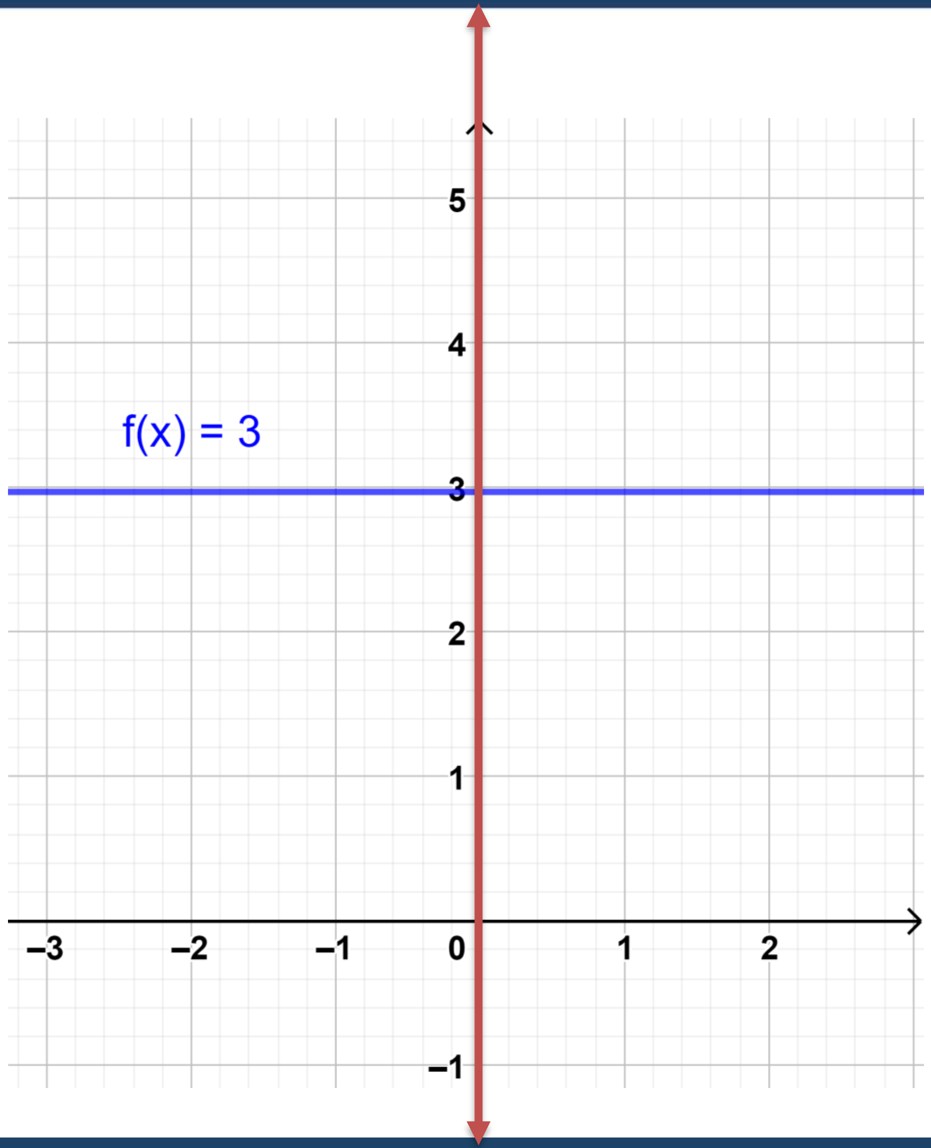


Types of Fun. & Graph (3/18)

Constant Function (3/3):

Zeroth degree: $f(x) = a$

If $a = 3$



Even

$f(-x) = f(x)$

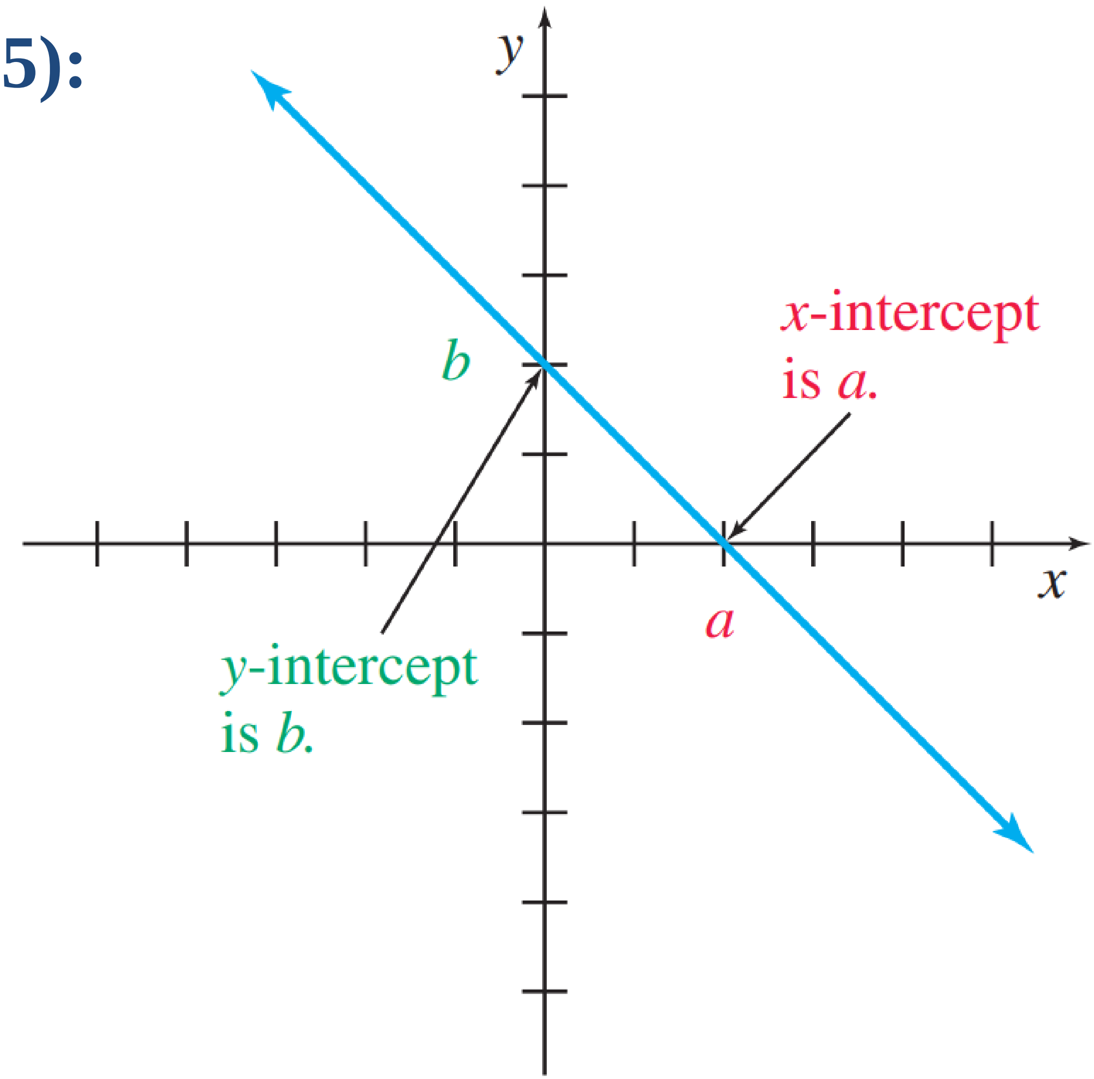
Linear Function (1/15):

First degree:

$$f(x) = mx + b$$

Set $x = 0$ to find b

Set $y = 0$ to find a

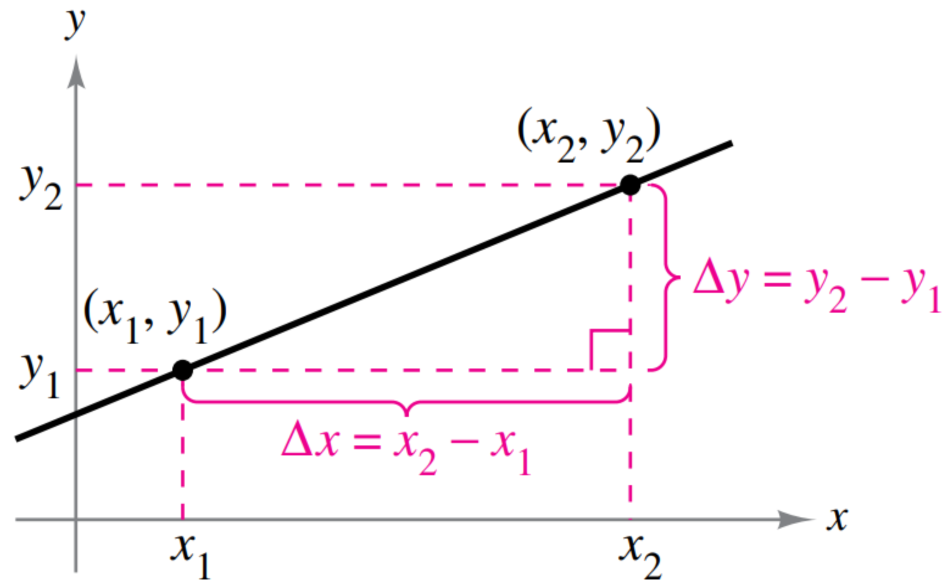


Linear Function (2/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.



$$m = \frac{\text{change of } y}{\text{change of } x} = \frac{\Delta y}{\Delta x}$$

$\Delta y = y_2 - y_1 = \text{change in } y$

$\Delta x = x_2 - x_1 = \text{change in } x$

The rate of change of y with respect to x

$x_1 \neq x_2$

Linear Function (3/15):

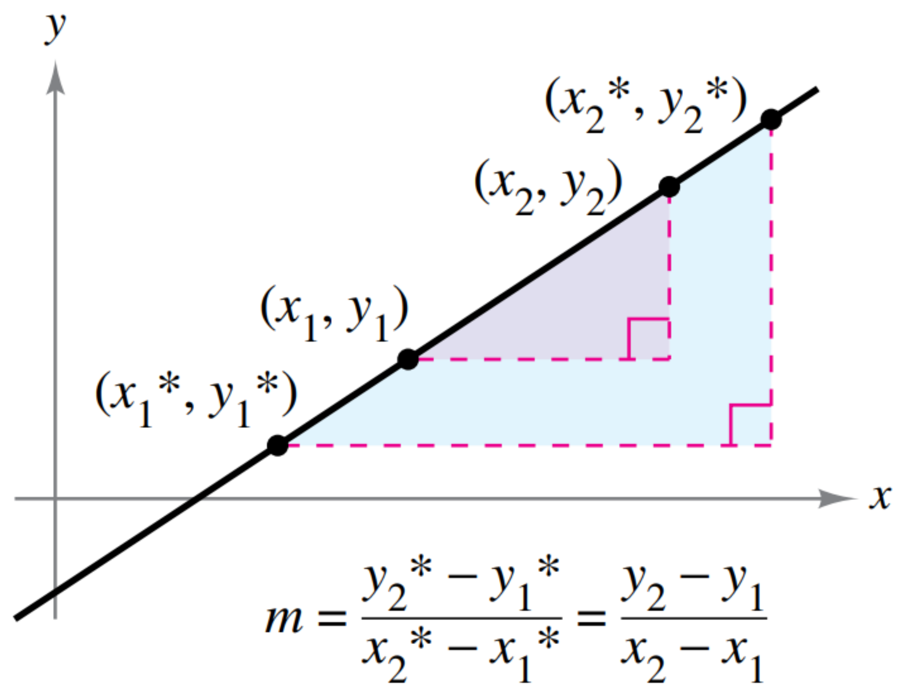
First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m = \frac{\text{change of } y}{\text{change of } x} = \frac{\Delta y}{\Delta x}$$

The rate of change is **constant**



Any two points on a nonvertical line can be used to determine its slope.

Linear Function (4/15):

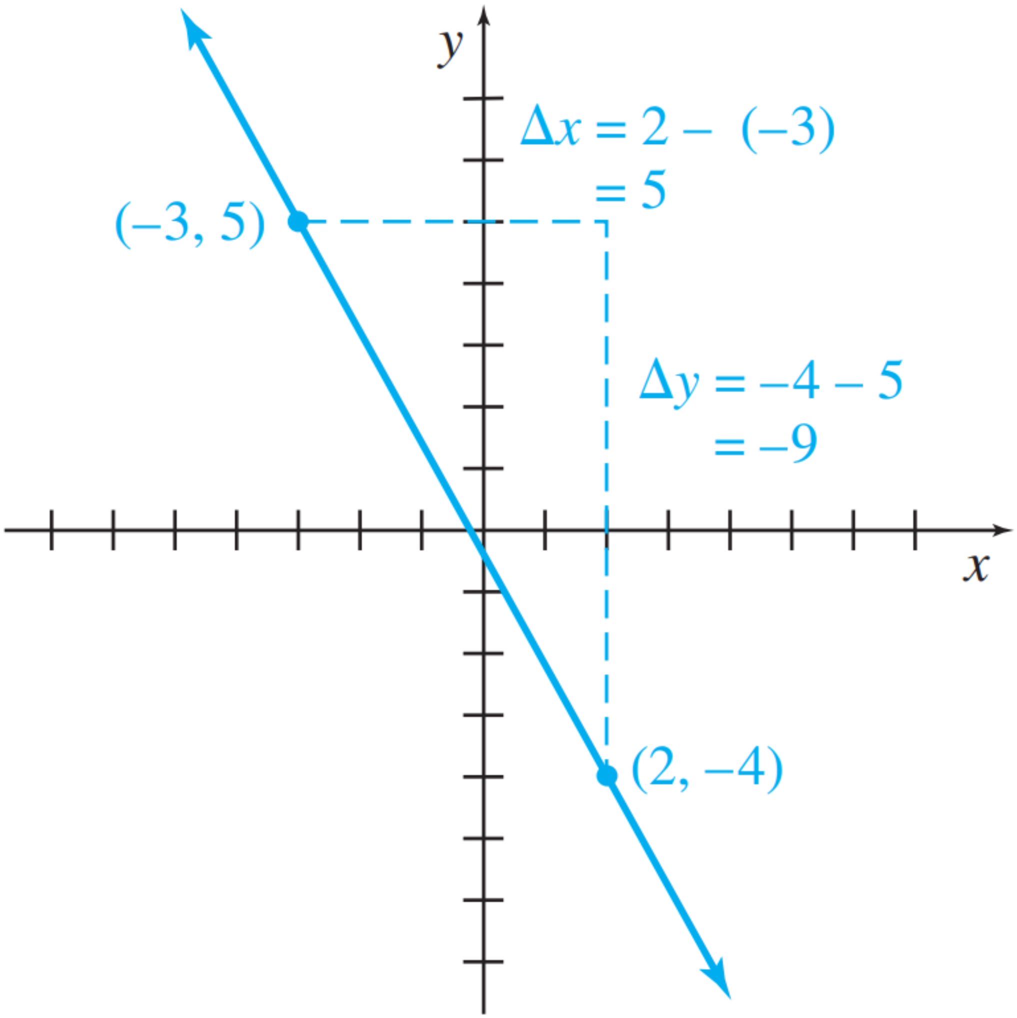
First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m = \frac{\text{change of } y}{\text{change of } x} = \frac{\Delta y}{\Delta x}$$

$$m = -9/5$$



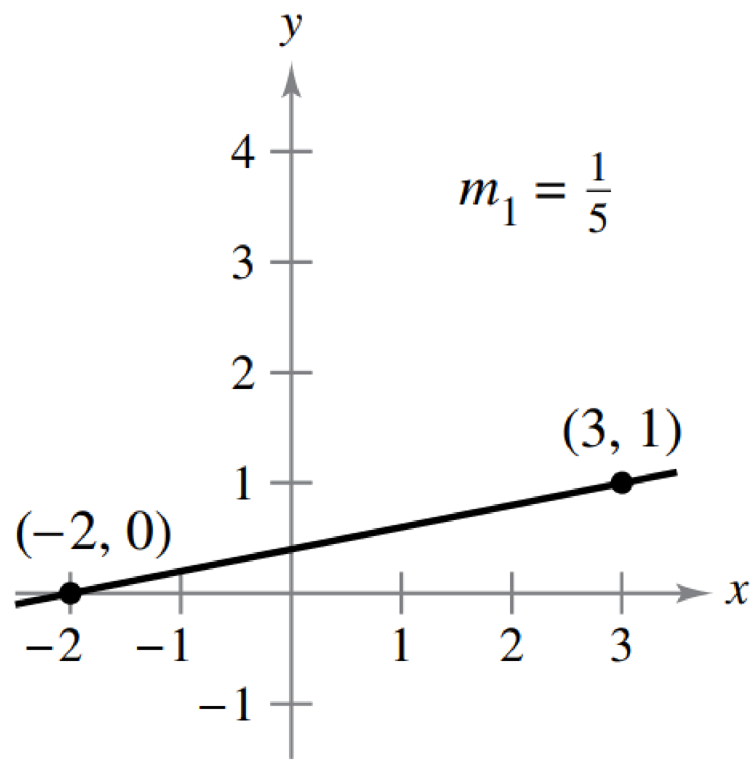
Linear Function (5/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m > 0$$



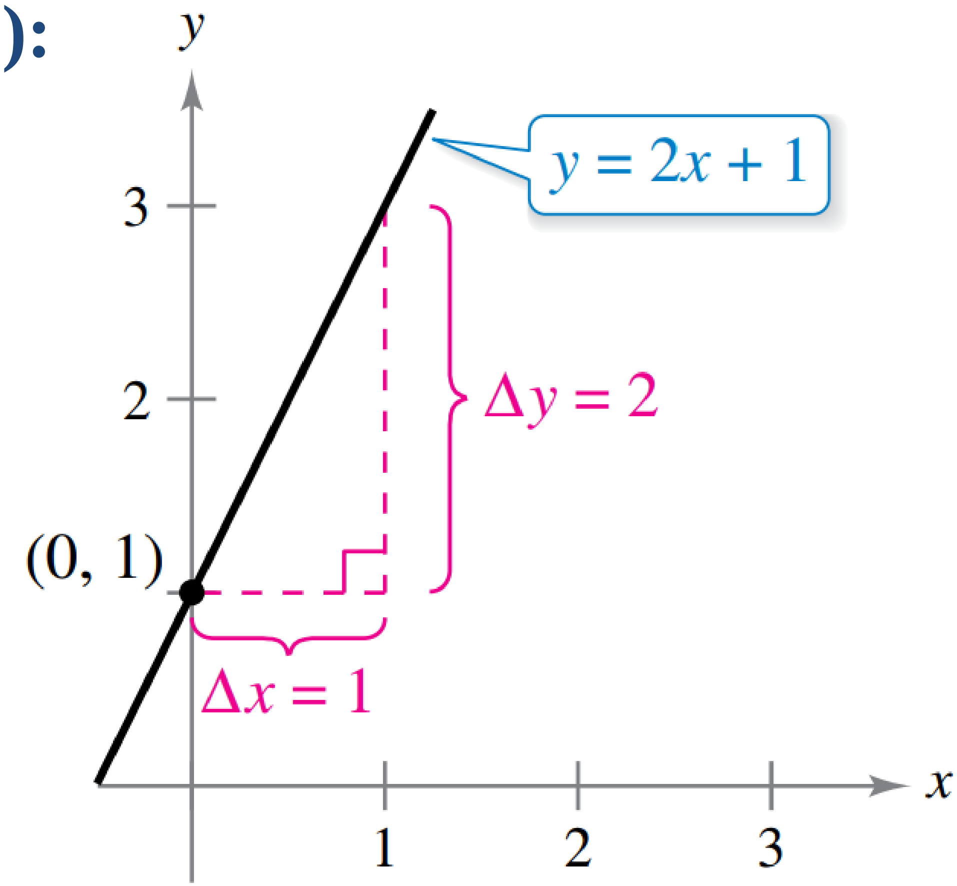
If m is positive, then the line rises from left to right.

Types of Fun. & Graph (4/18)

Linear Function (6/15):

First degree:

$$f(x) = 2x + 1$$



$m = 2$; line rises

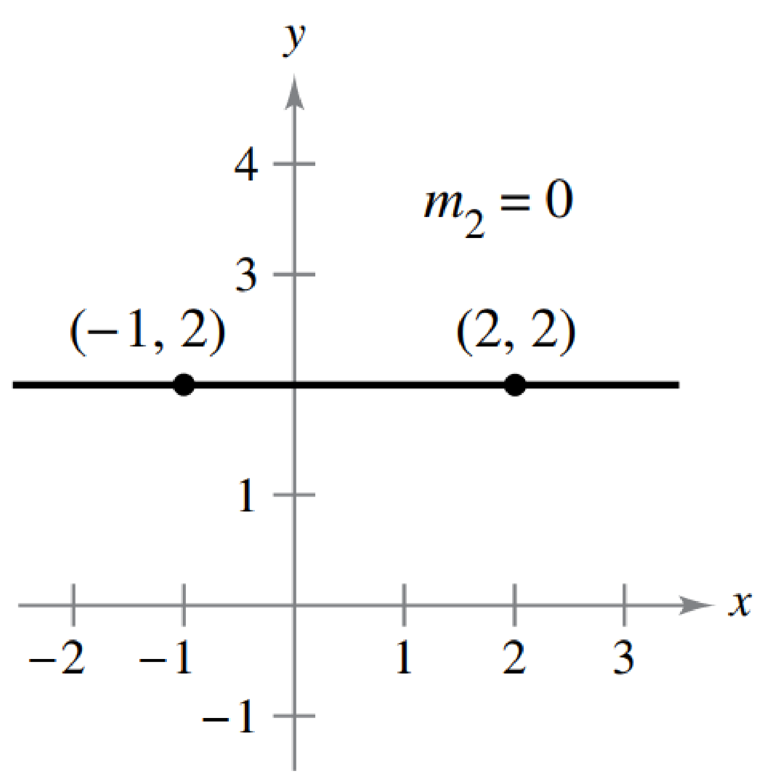
Linear Function (7/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m = 0$$



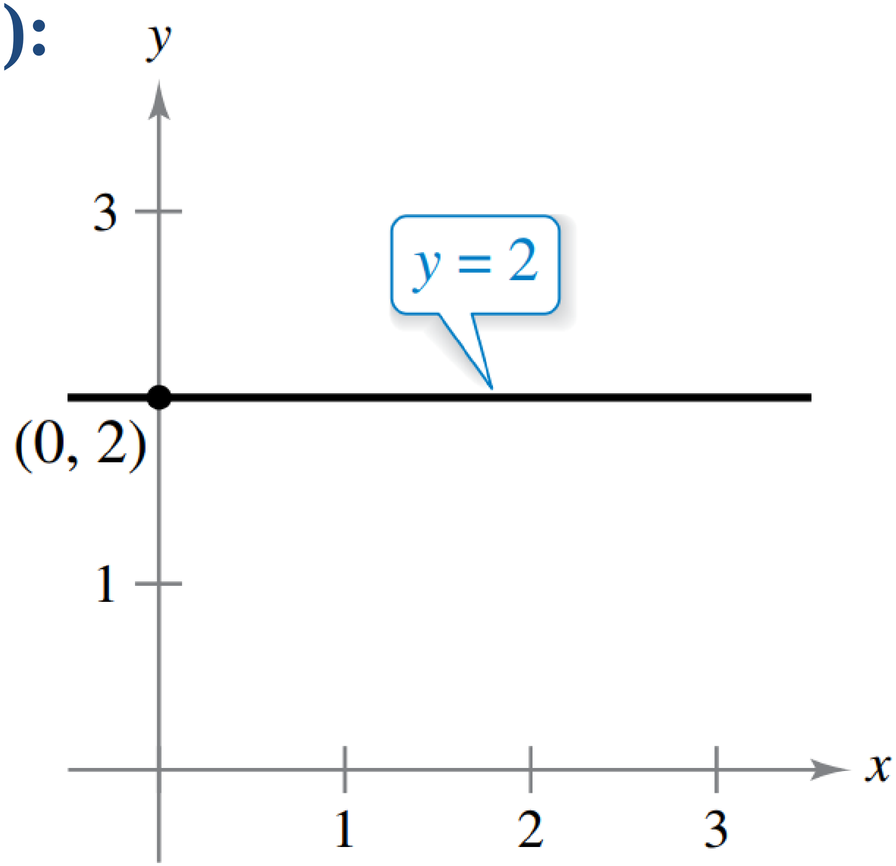
If m is zero, then the line is horizontal.

Linear Function (8/15):

First degree:

$$f(x) = mx + b$$

If $m = 0$, $b = 2$



$m = 0$; line is horizontal

Types of Fun. & Graph (4/18)

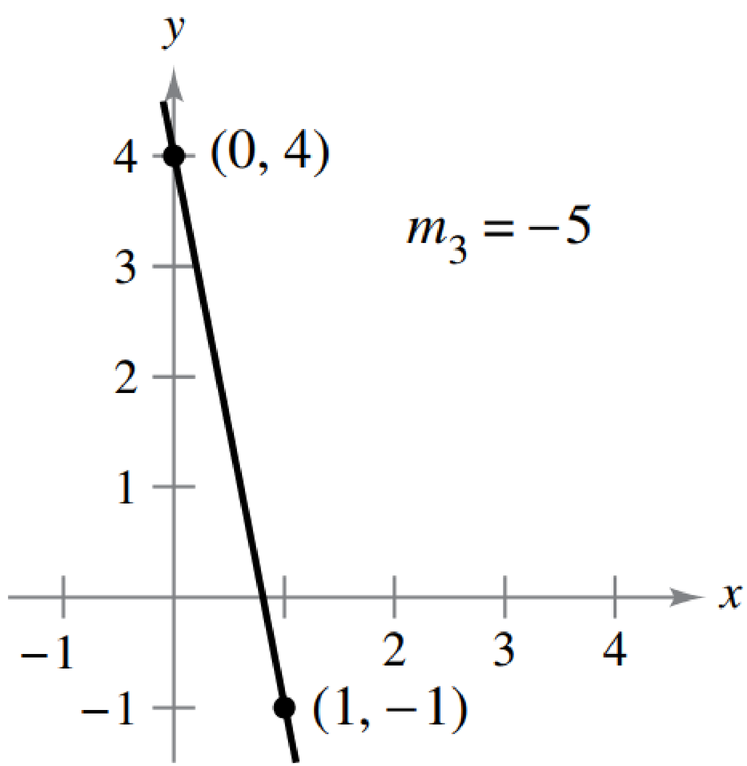
Linear Function (9/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$m < 0$



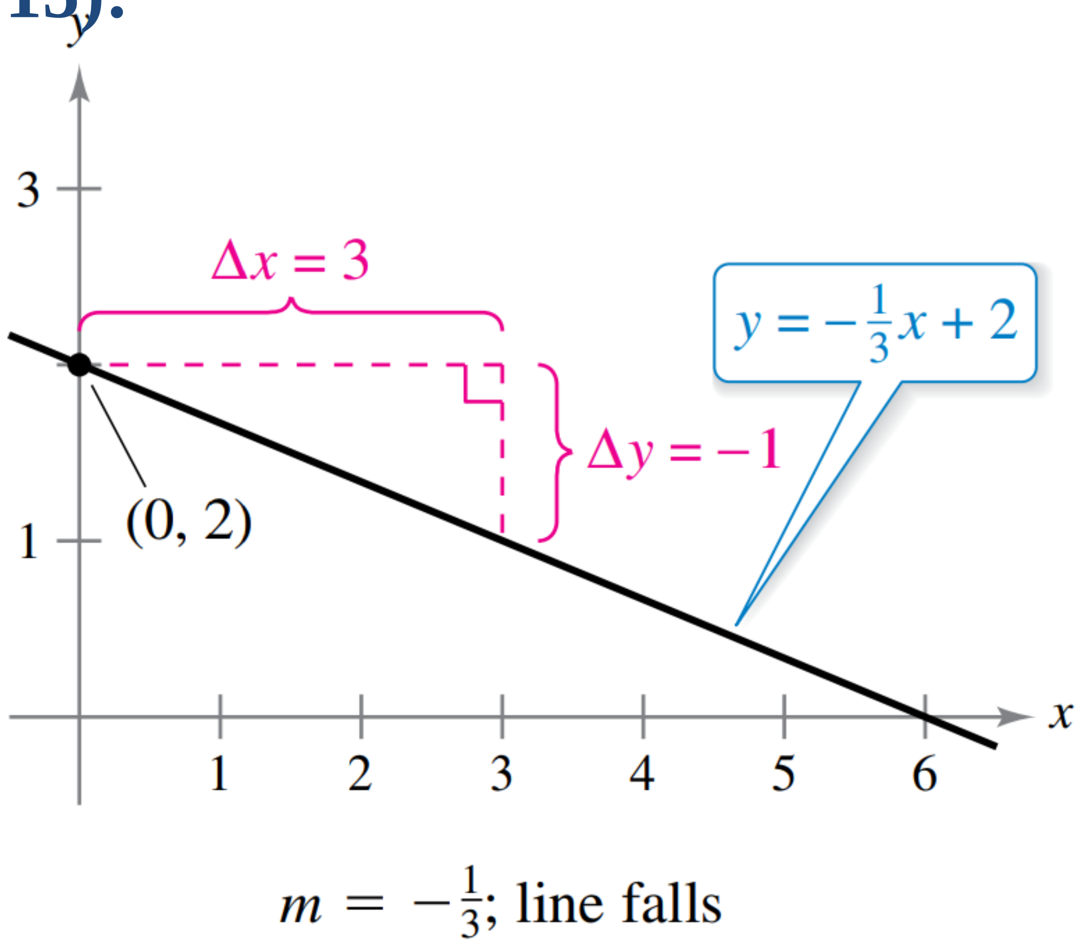
If m is negative, then the line falls from left to right.

Types of Fun. & Graph (4/18)

Linear Function (10/15):

First degree:

$$f(x) = -\frac{1}{3}x + 2$$



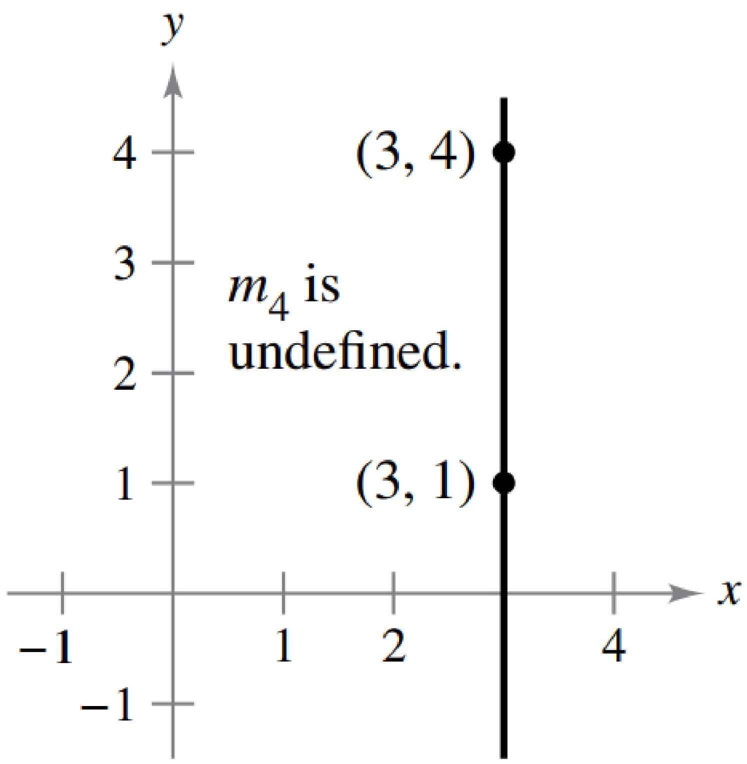
Linear Function (11/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

m is undefined



If m is undefined, then the line is vertical.

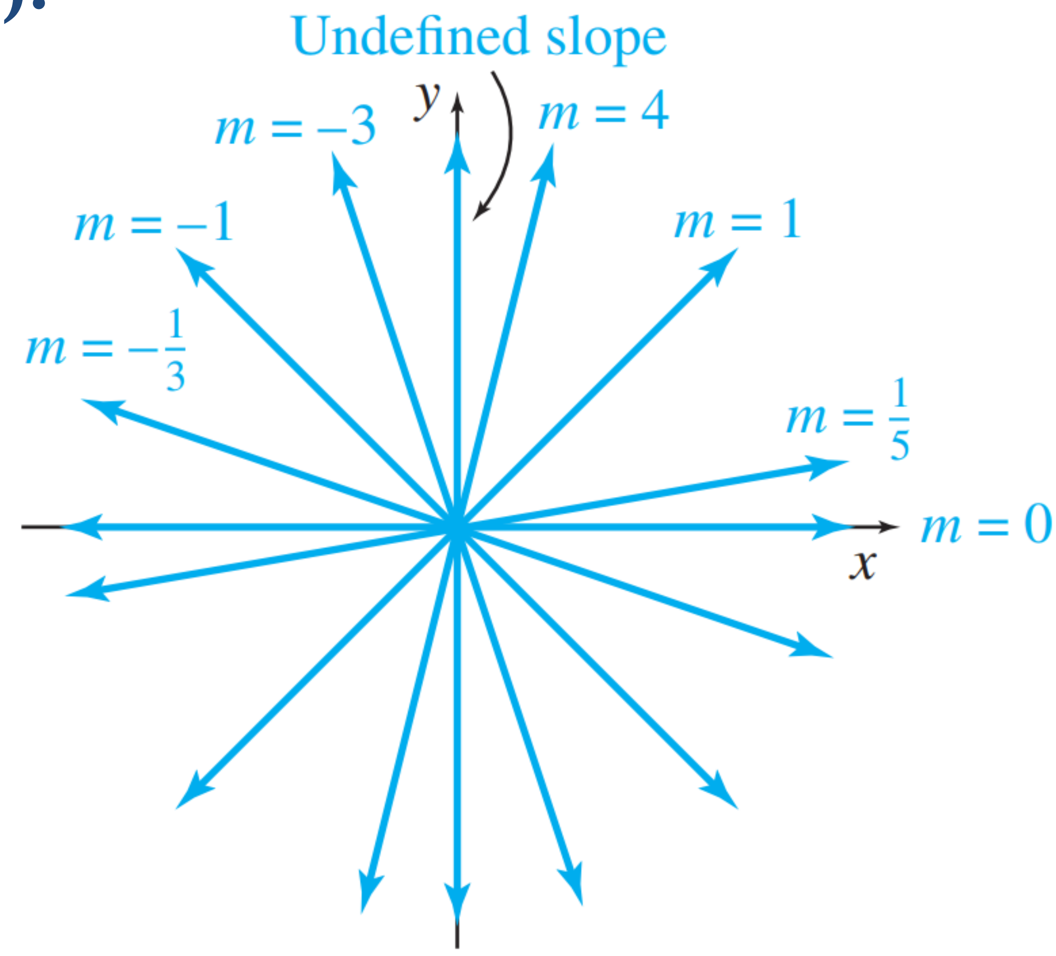
Linear Function (12/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m = \frac{\text{change of } y}{\text{change of } x} = \frac{\Delta y}{\Delta x}$$



Linear Function (13/15):

$$f(x) = 2x - 4$$

$$f(x) = mx + b$$

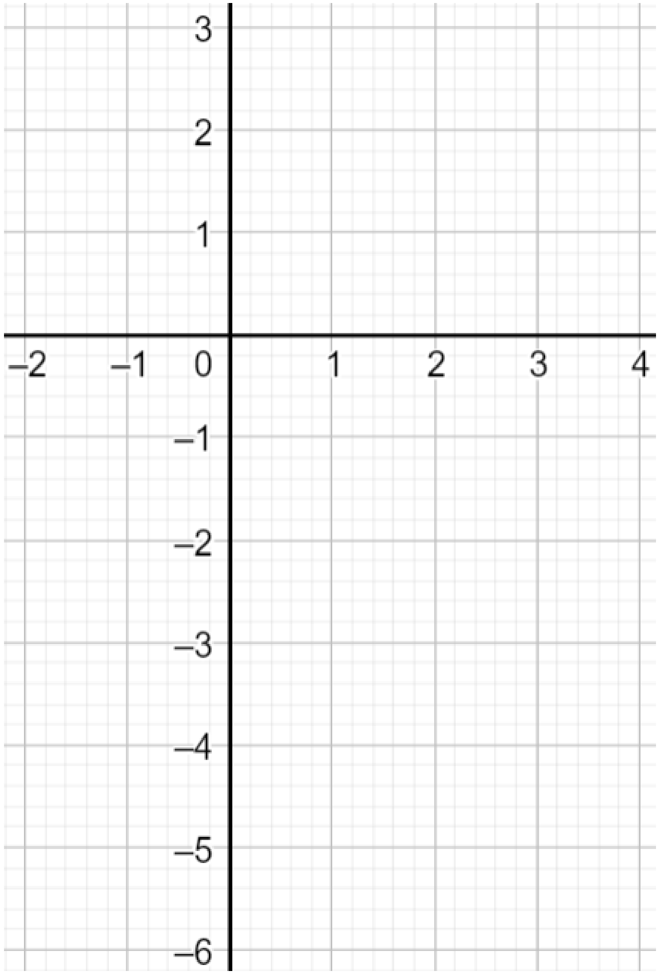
$$m = 2$$

$$b = -4$$

$\underline{y\_intercept}$

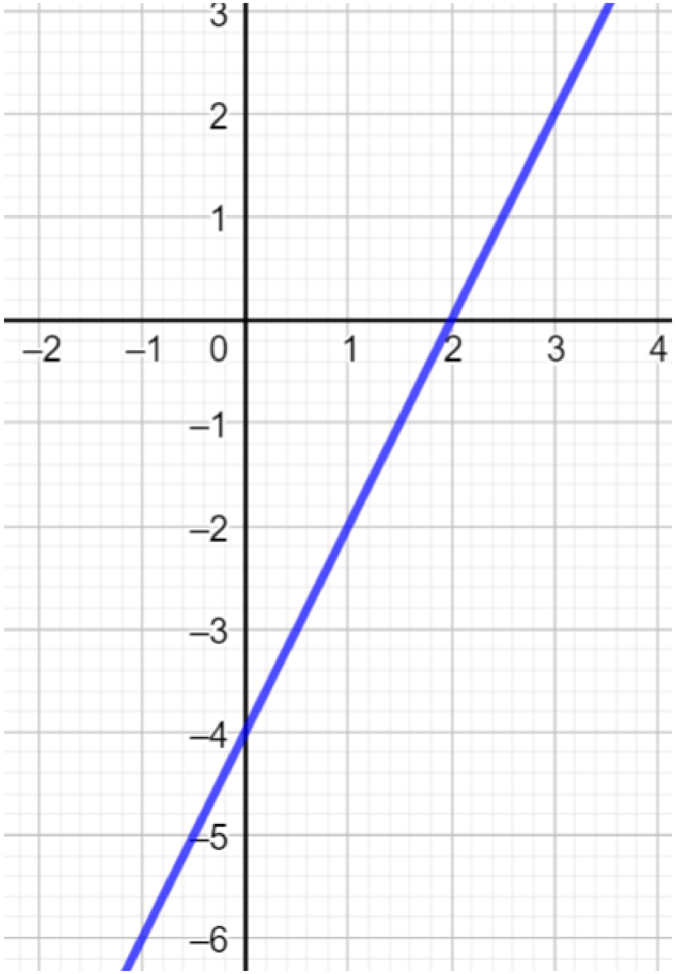
$\underline{x\_intercept}$

$$2x - 4 = 0 \rightarrow x = 2$$



Linear Function (13/15):

$$f(x) = 2x - 4$$



$$f(x) = mx + b$$

$$m = 2$$

$$b = -4$$ <u>$y\_intercept$</u>

<u>$x\_intercept$</u>

$$2x - 4 = 0 \rightarrow x = 2$$

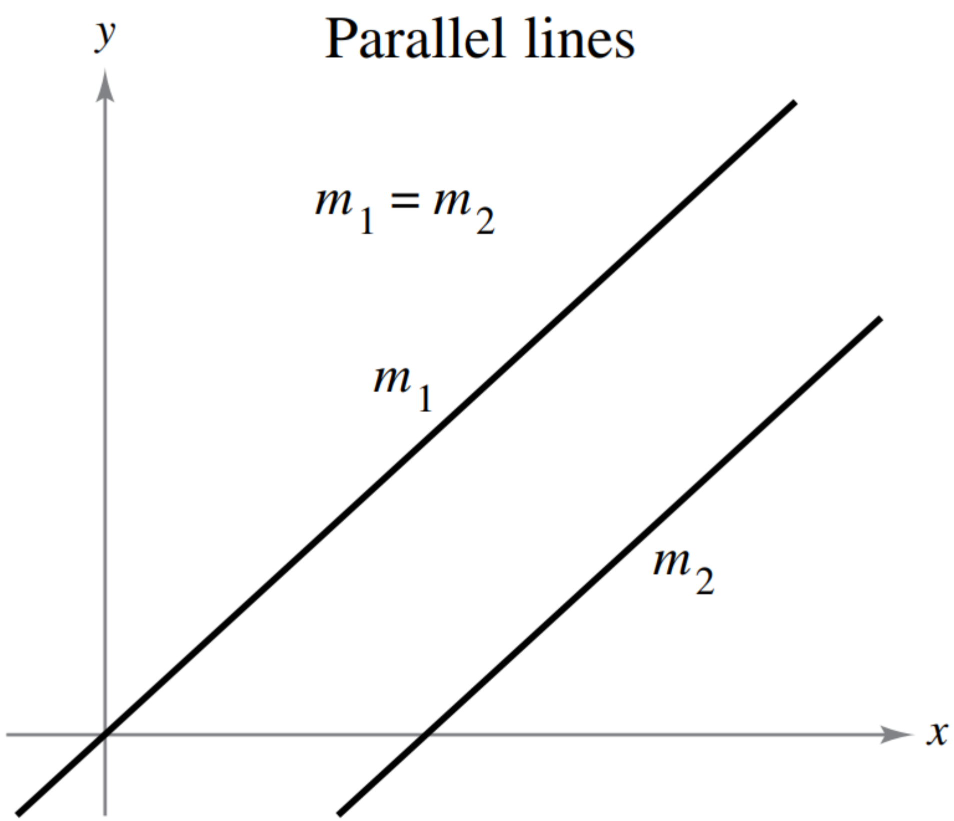
Linear Function (14/15):

First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m_1 = m_2$$



Linear Function (15/15):

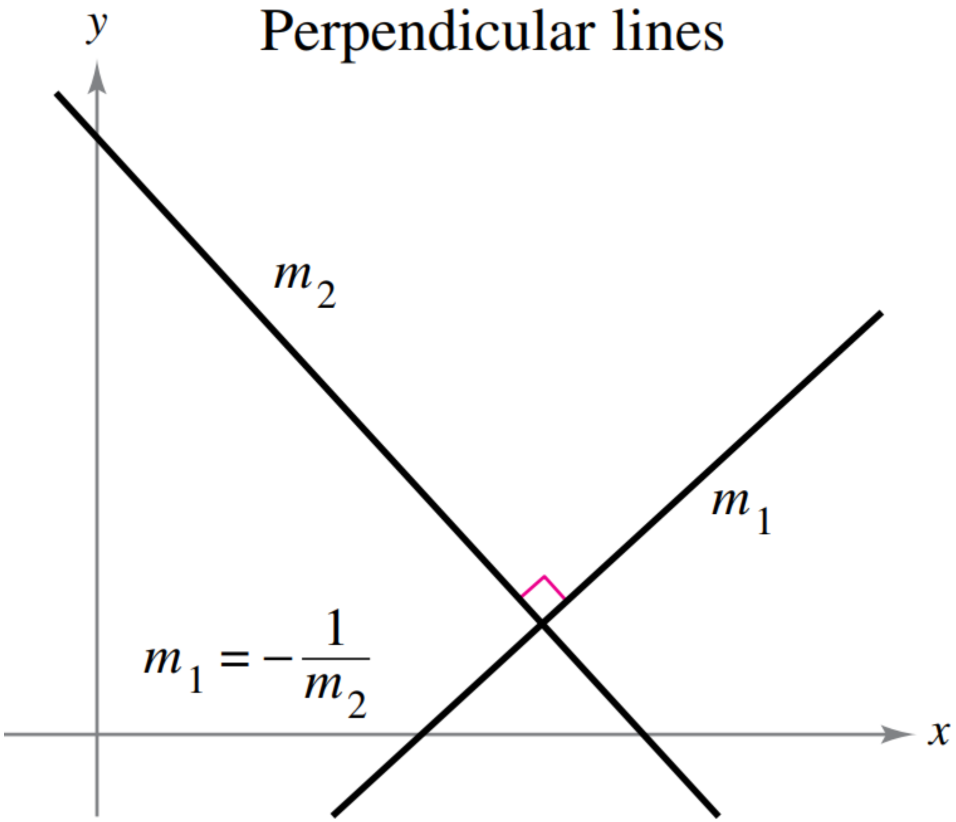
First degree:

$$f(x) = mx + b$$

The number m is called the **slope** of the line.

$$m_1 = -\frac{1}{m_2}$$

Or $m_1 m_2 = -1$



Types of Fun. & Graph (5/18)

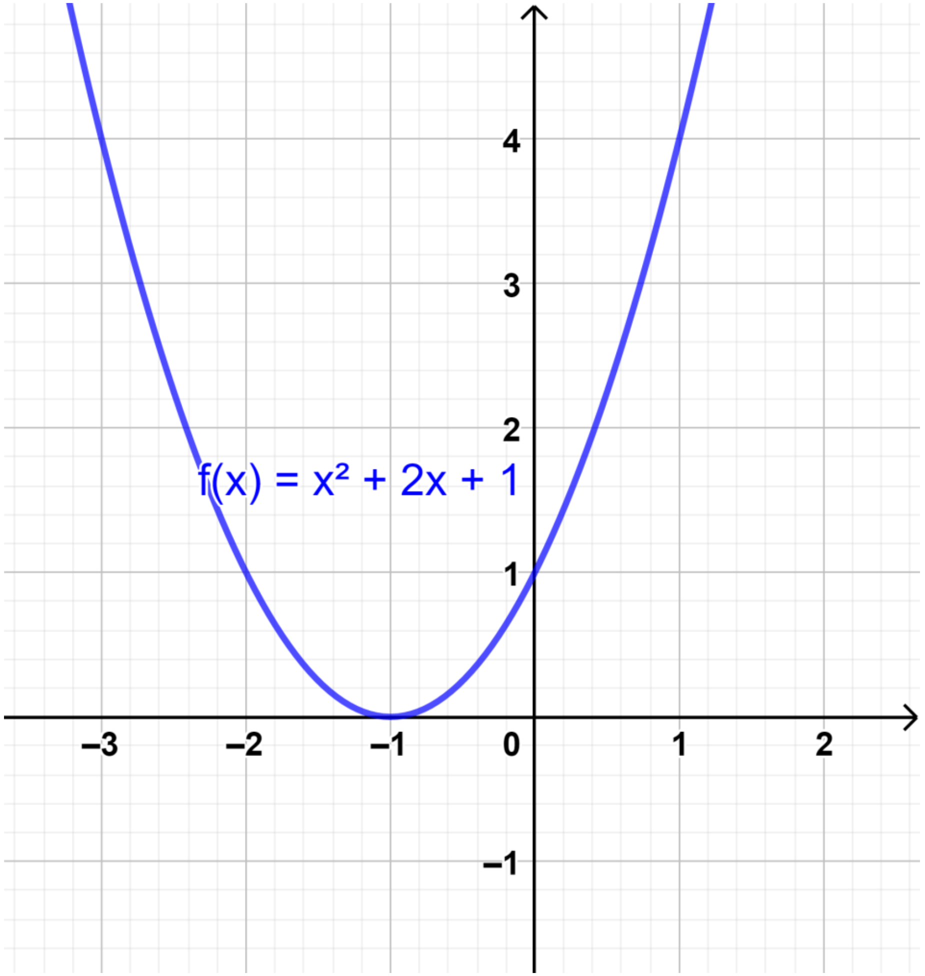
Vertical Parabola (1/3):

Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \; c = 1$



Types of Fun. & Graph (5/18)

Vertical Parabola (2/3):

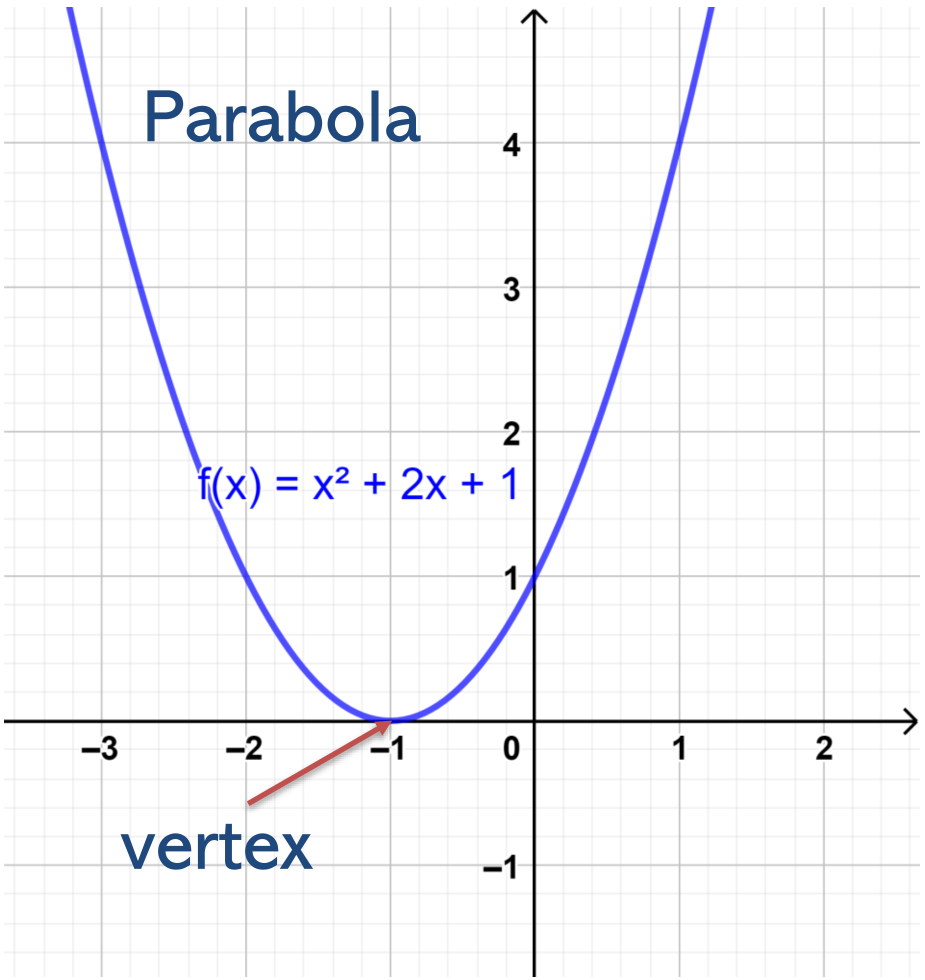
Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \ c = 1$

The parabola opens upward if $a > 0$ and downward if $a < 0$

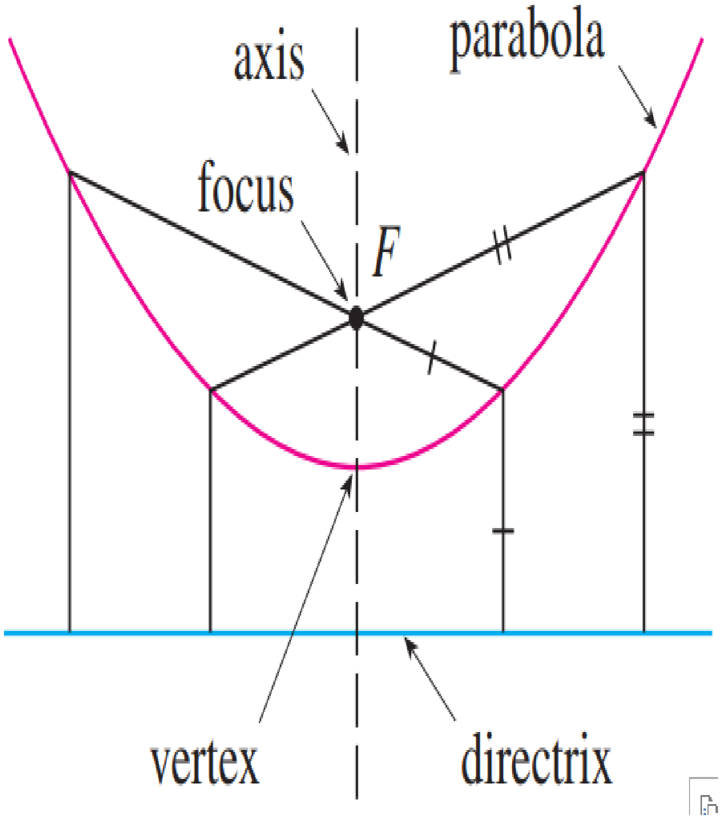


Vertical Parabola (3/3):

If a quadratic equation is given in the form $ax^2 + bx + c$, we can use completing the square to rewrite it in the form

$$y = a(x - \boldsymbol{h})^2 + \boldsymbol{k}$$

In this form, we can identify the vertex as $(\boldsymbol{h}, \boldsymbol{k})$



The parabola opens upward if $a > 0$ and downward if $a < 0$

Completing Square (1/2):

$$y = a(x - h)^2 + k$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 + 2ax = (x + a)^2 - a^2$$

If a quadratic equation is given in the form

$y = x^2 - 4x + 3$, then

$$y = \left((x - 2)^2 - 4\right) + 3$$

$$y = (x - 2)^2 - 1 \quad \rightarrow \quad vertex = (2, -1)$$

The parabola opens upward because $a = 1 > 0$

Types of Fun. & Graph (6/18)

Completing Square (2/2):

$$y = a(x - h)^2 + k$$

$$\boldsymbol{x^2 + 2ax + a^2 = (x + a)^2}$$

$$\boldsymbol{x^2 + 2ax = (x + a)^2 - a^2}$$

If a quadratic equation is given in the form

$y = 2x^2 + 4x + 7$, then

$$y = 2(x^2 + 2x) + 7 \quad \rightarrow \quad y = 2\left((x + 1)^2 - 1\right) + 7$$

$$y = 2(x + 1)^2 + 5 \rightarrow \quad vertex = (-1, 5)$$

The parabola opens upward because $a = 2 > 0$

Another Method:

If a quadratic equation is given in the form

$$y = x^2 - 4x + 3$$

$$y = ax^2 + bx + c,$$

then $a = 1,\ b = -4,\ c = 3$

$$h = \boxed{\frac{-b}{2a}} = \frac{4}{2} = 2$$

$$\therefore vertex = (2, -1)$$

$$k = (2)^2 - 4(2) + 3 = -1$$

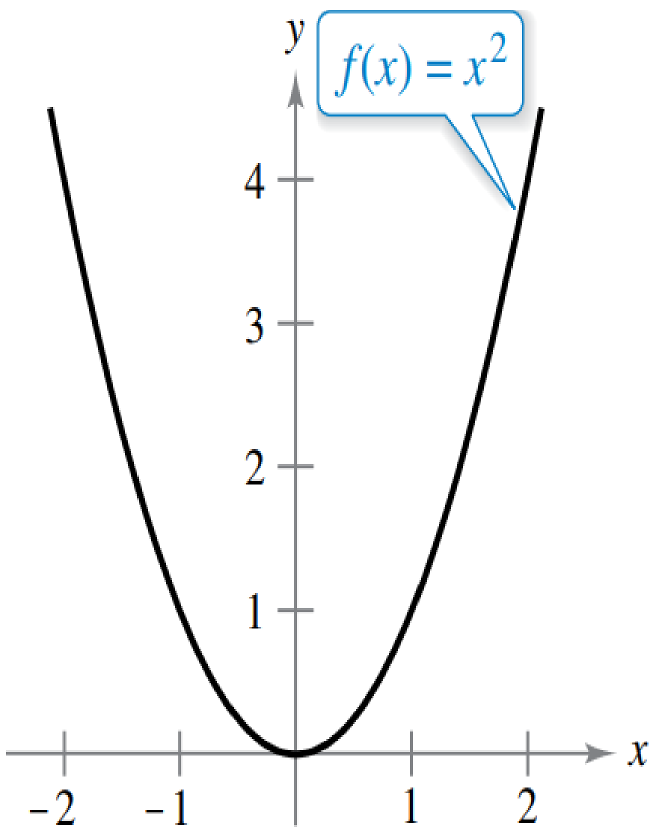
Example 1:

$y = x^2$, we can use completing the square to rewrite it in the form

$$y = 1(x - \mathbf{0})^2 + \mathbf{0}$$

$$y = a(x - \boldsymbol{h})^2 + \boldsymbol{k}$$

In this form, we can identify the vertex as $(\mathbf{0}, \mathbf{0})$



The parabola opens upward because $a > 0$

Types of Fun. & Graph (8/18)

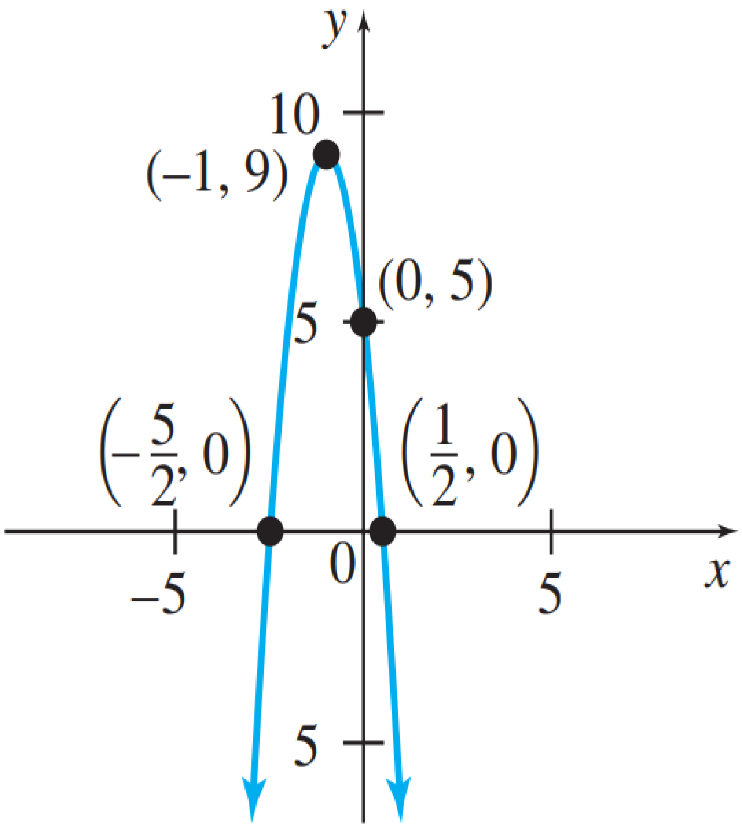
Example 2:

$y = -4x^2 - 8x + 5$, we can use completing the square to rewrite it in the form

$$h = \frac{-b}{2a} = \frac{8}{-8} = -1$$

$$k = (-4)(-1)^2 - 8(-1) + 5 = 9$$

So, the vertex is $(\mathbf{-1}, \mathbf{9})$



The parabola opens downward because $a < 0$

Example 3:

Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \ c = 1$

$$f(x) = x^2 + 2x + 1$$

Types of Fun. & Graph (9/18)

Example 3:

Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2,\ c = 1$

$$f(x) = x^2 + 2x + 1$$

$$h = -\frac{b}{2a} = -\frac{2}{2} = -1$$

$$k = f(h) = f(-1)$$

$$k = 0$$

$$\therefore vertex = (-1, 0)$$

Types of Fun. & Graph (9/18)

Example 3:

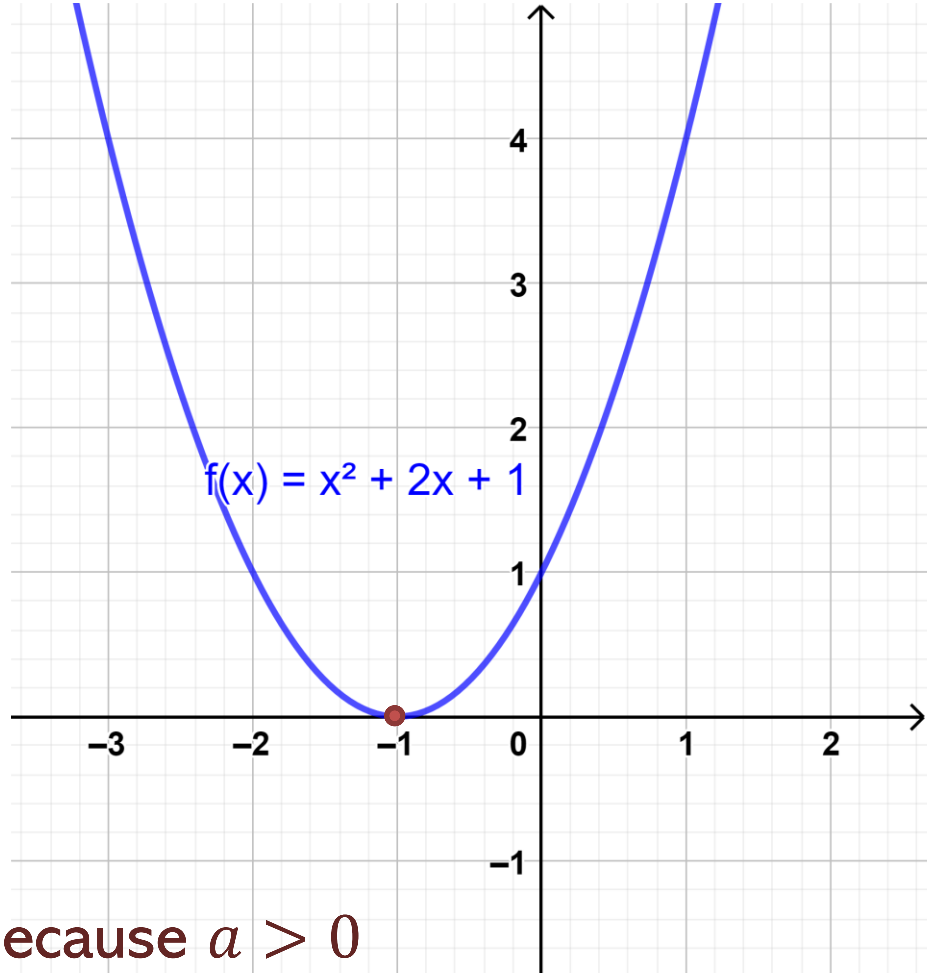
Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \ c = 1$

$$f(x) = x^2 + 2x + 1$$



The parabola opens upward because $a > 0$

Types of Fun. & Graph (9/18)

Example 3:

Quadratic function

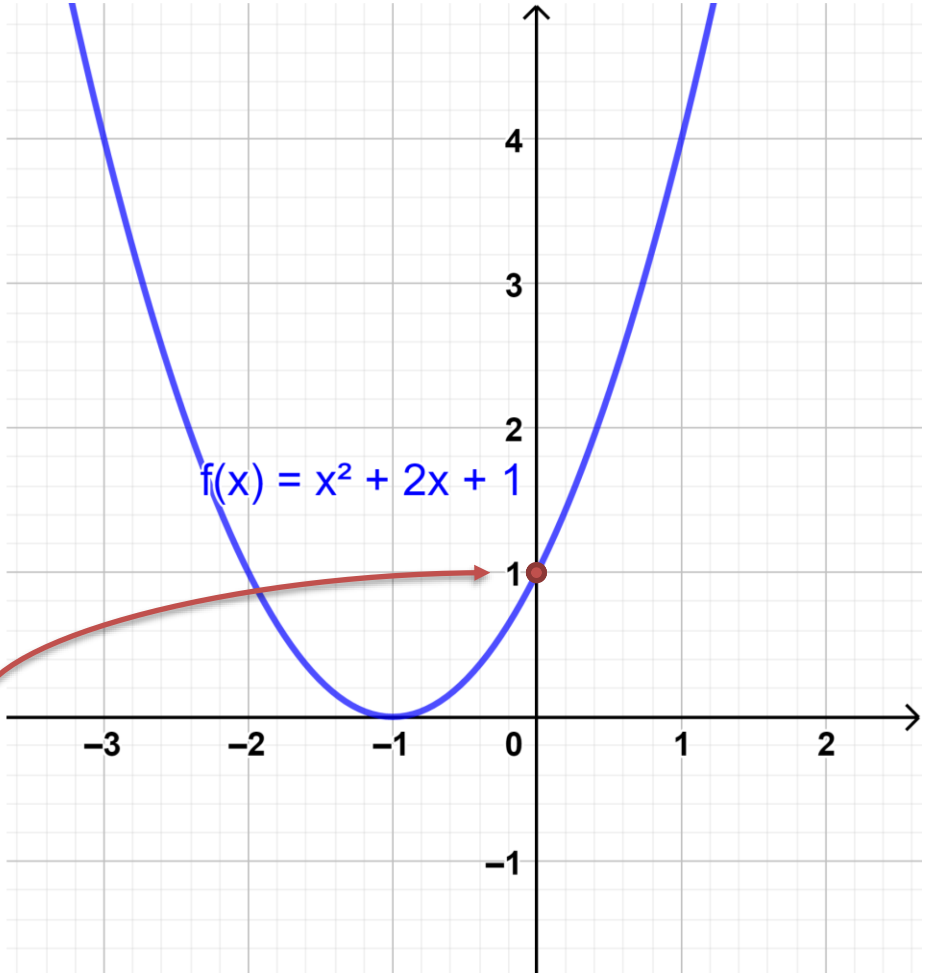
Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \ c = 1$

$$f(x) = x^2 + 2x + 1$$

$$f(0) = 1$$



Types of Fun. & Graph (9/18)

Example 3:

Quadratic function

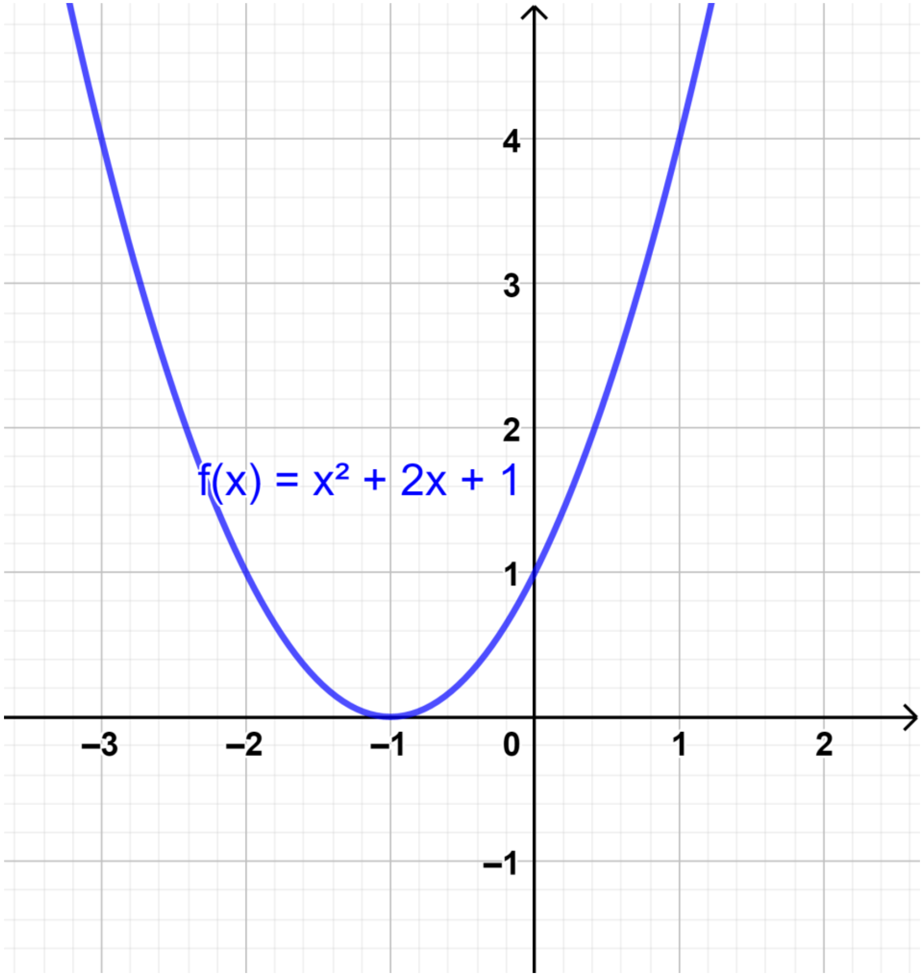
Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \ c = 1$

Domain = ? ?

Range = ? ?



Types of Fun. & Graph (9/18)

Example 3:

Quadratic function

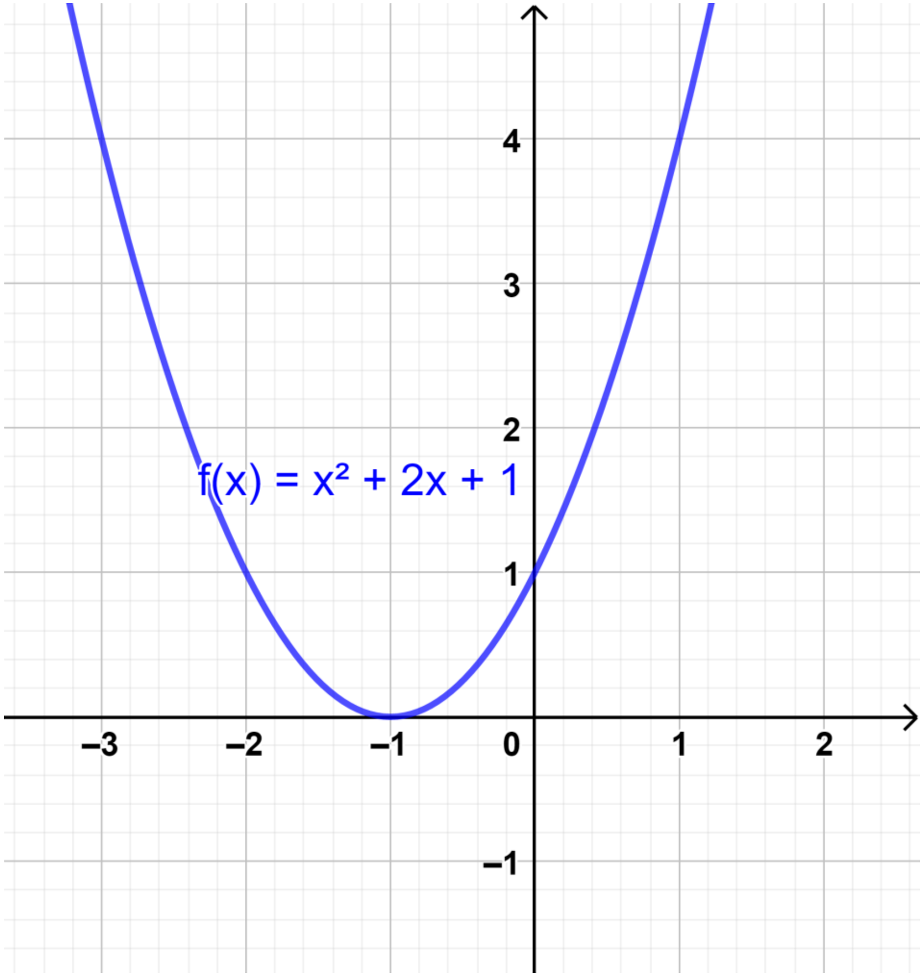
Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2,\ c = 1$

Domain = $\mathbb{R}$

Range = $[0, \infty)$



Types of Fun. & Graph (9/18)

Example 3:

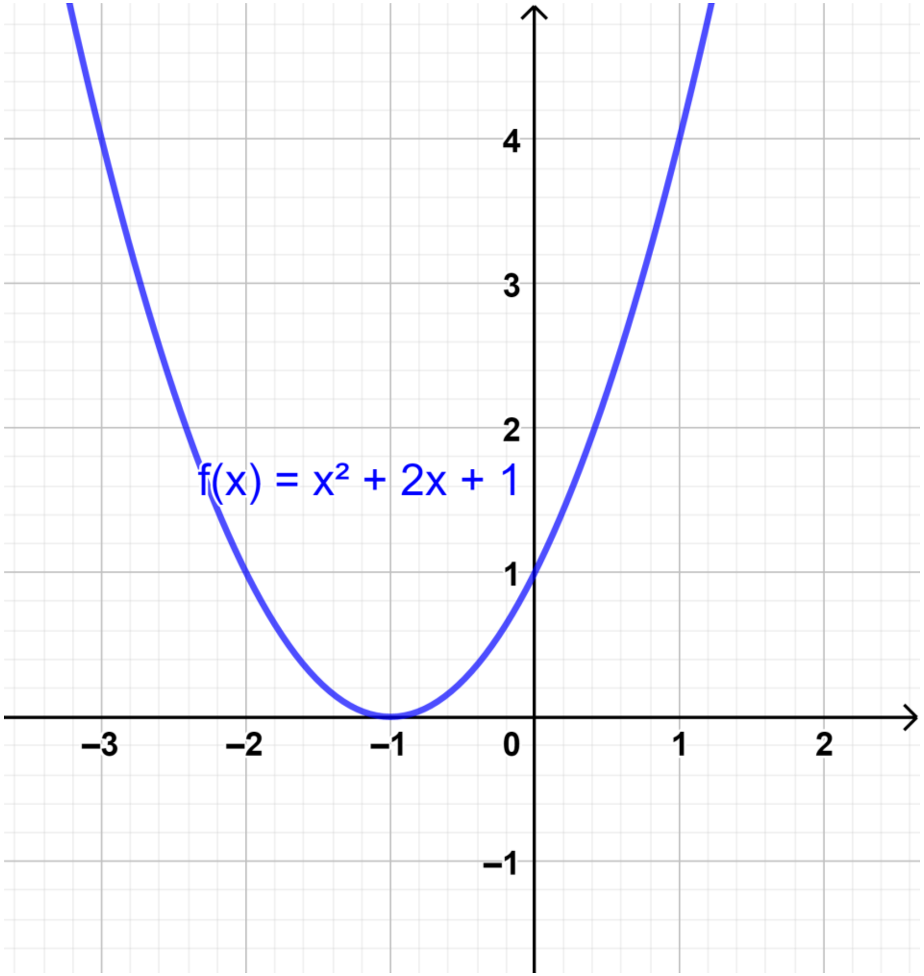
Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2, \; c = 1$

Odd or Even??



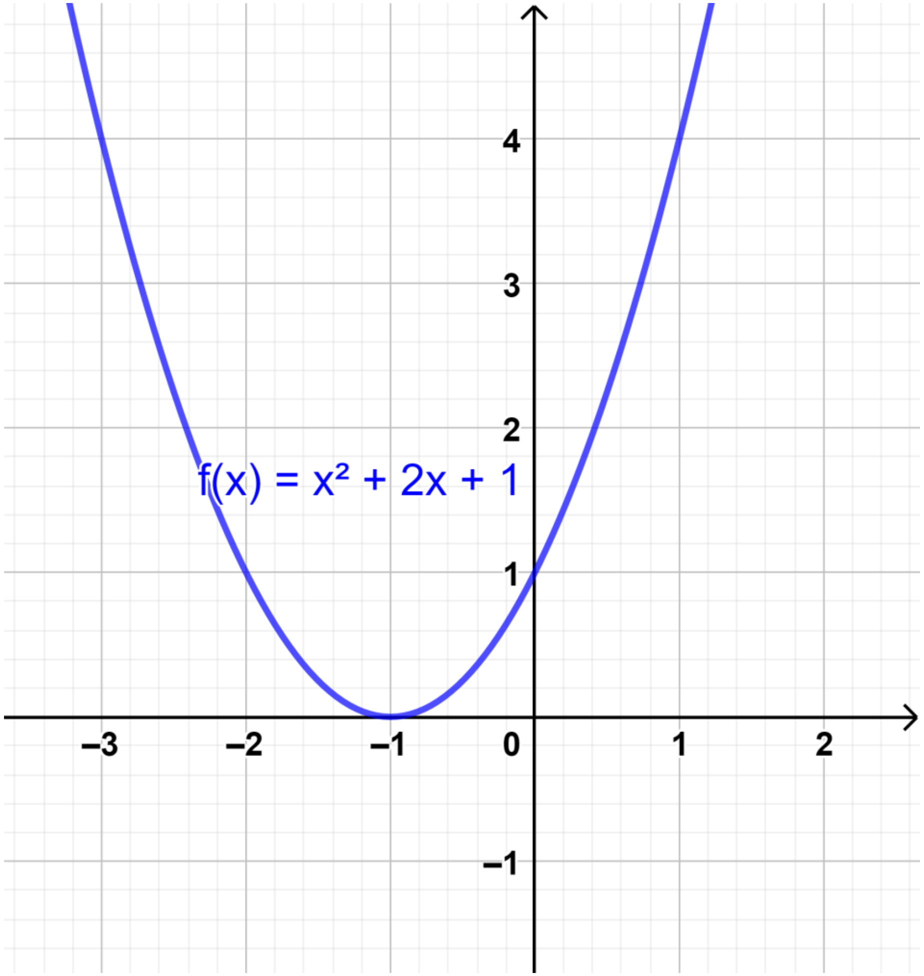
Example 3:

Quadratic function

Second degree:

$$f(x) = ax^2 + bx + c$$

If $a = 1, b = 2,\ c = 1$



Not odd

Not Even

Types of Fun. & Graph (10/18)

Examples of Polynomial Functions (1/8):

$$f(x) = x^2 - 2x + 3$$

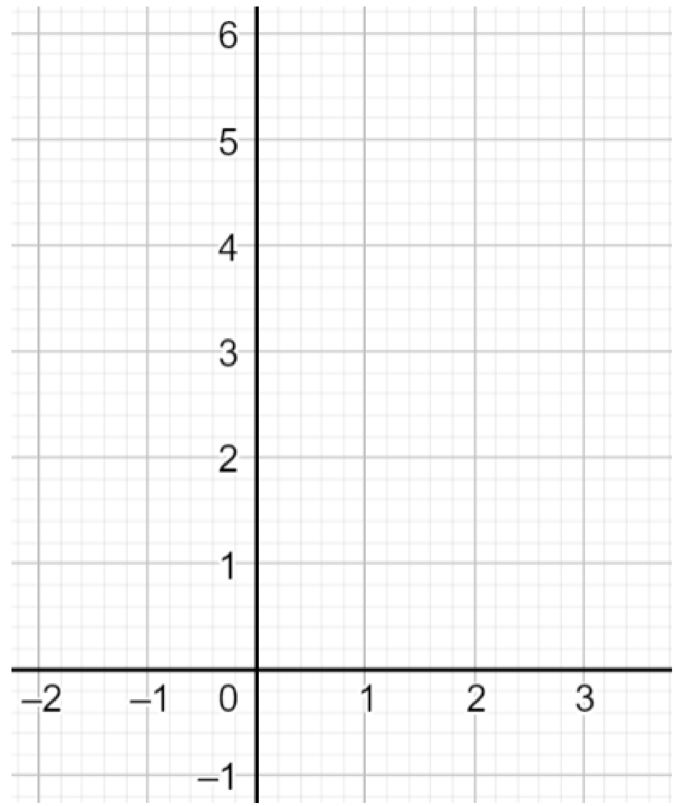
$$f(x) = ax^2 + bx + c$$

$$a = 1, \quad b = -2, \quad c = 3$$

$$h = -\frac{b}{2a} = \frac{-(-2)}{2} = 1$$

$$k = f(1) = (1)^2 - 2(1) + 3 = 2$$

$$\therefore vertex = (1, 2)$$



Examples of Polynomial Functions (2/8):

$$f(x) = x^2 - 2x + 3$$

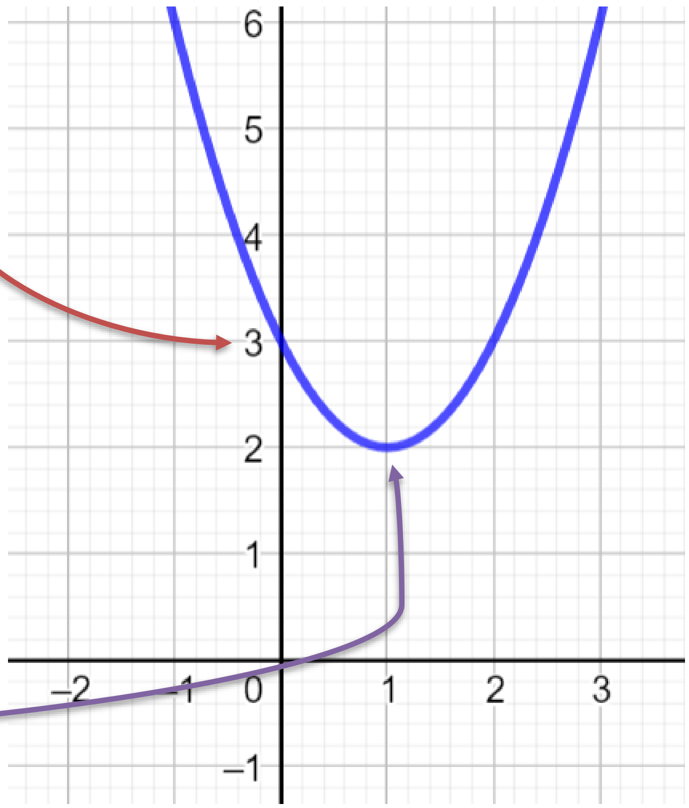
$$f(x) = ax^2 + bx + c$$

$$a = 1, \quad b = -2, \quad c = 3$$

$$h = -\frac{b}{2a} = \frac{-(-2)}{2} = 1$$

$$k = f(1) = (1)^2 - 2(1) + 3 = 2$$

$$\therefore vertex = (1, 2)$$



Examples of Polynomial Functions (3/8):

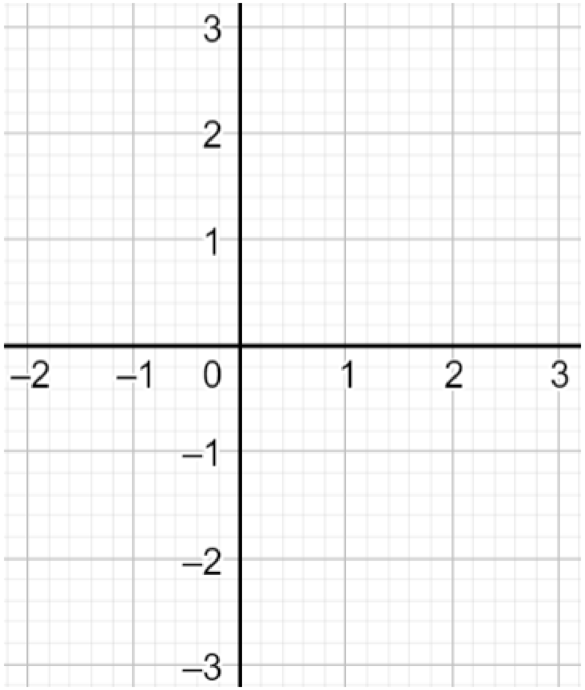
$$f(x) = 2x^2 - 4x$$

$$a = 2, \quad b = -4, \quad c = 0$$

$$h = -\frac{b}{2a} = \frac{-(-4)}{2(2)} = 1$$

$$k = f(1) = 2(1)^2 - 4(1) = -2$$

$$\therefore vertex = (1, -2)$$



Examples of Polynomial Functions (4/8):

$f(x) = 2x^2 - 4x$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Real Roots:

$2x^2 - 4x = 0$

$a = 2, \quad b = -4, \quad c = 0$

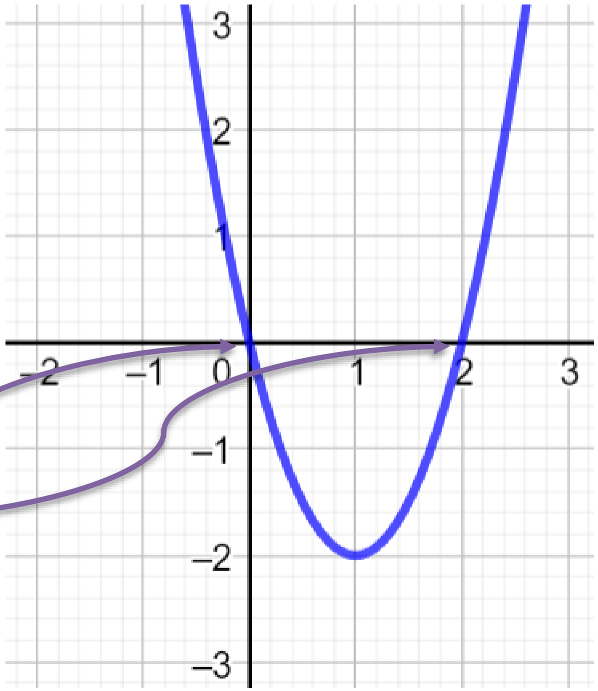
1) *Two Real Roots*, If $(b^2 - 4ac) > 0$
2) *One Real Root (Double root)*, If $(b^2 - 4ac) = 0$
3) *No Real Roots*, If $(b^2 - 4ac) < 0$

From quadratic formula

Two Real roots: $x_1 = 0, \quad x_2 = 2$

Examples of Polynomial Functions (5/8):

$$f(x) = 2x^2 - 4x$$



Real Roots:

$2x^2 - 4x = 0$

$a = 2, \;\; b = -4, \;\; c = 0$

<u>From quadratic formula</u>

Real roots: $x_1 = 0, \;\; x_2 = 2$

Examples of Polynomial Functions (6/8):

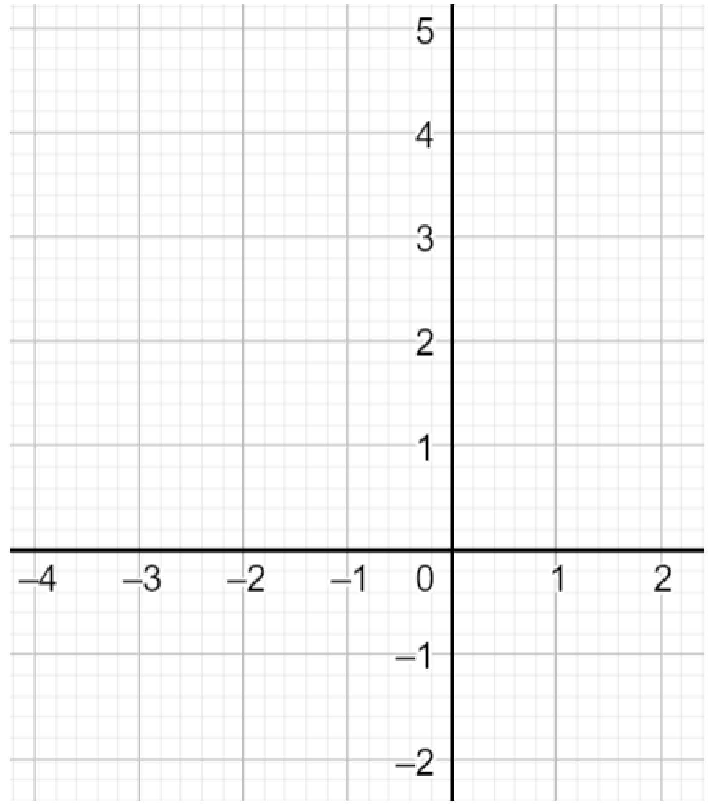
$$f(x) = -2x^2 - 4x + 2$$

$$a = -2, \quad b = -4, \quad c = 2$$

$$h = -\frac{b}{2a} = \frac{-(-4)}{2(-2)} = -1$$

$$k = f(-1) = -2(-1)^2 - 4(-1) + 2 = 4$$

$$\therefore \; vertex = (-1, 4)$$



Types of Fun. & Graph (12/18)

Examples of Polynomial Functions (7/8):

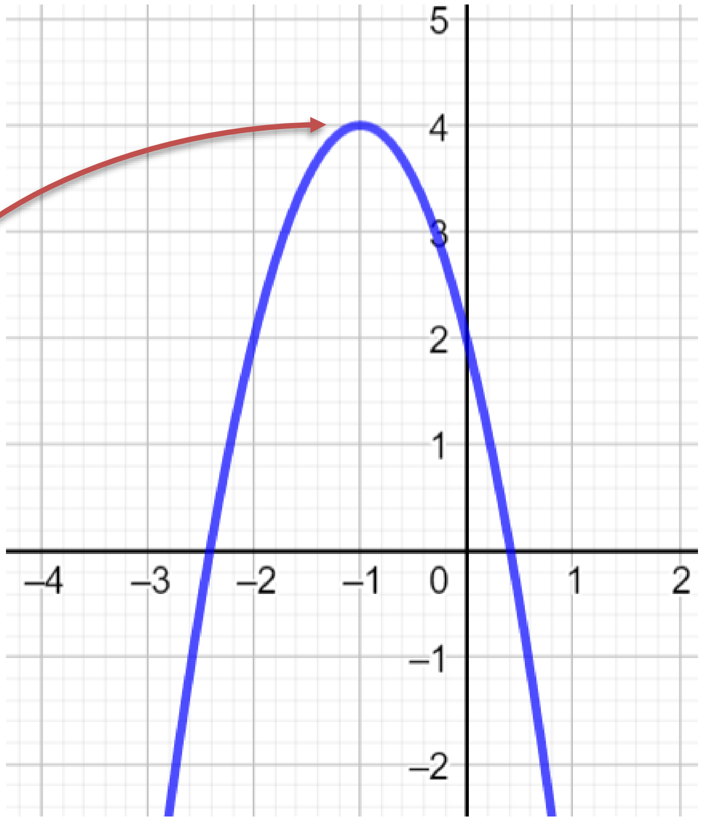
$$f(x) = -2x^2 - 4x + 2$$

$a = -2, \quad b = -4, \quad c = 2$

$$h = -\frac{b}{2a} = \frac{-(-4)}{2(-2)} = -1$$

$$k = f(-1) = -2(-1)^2 - 4(-1) + 2 = 4$$

$\therefore vertex = (-1, 4)$



Types of Fun. & Graph (12/18)

Examples of Polynomial Functions (8/8):

$$f(x) = -2x^2 - 4x + 2$$

Real Roots:

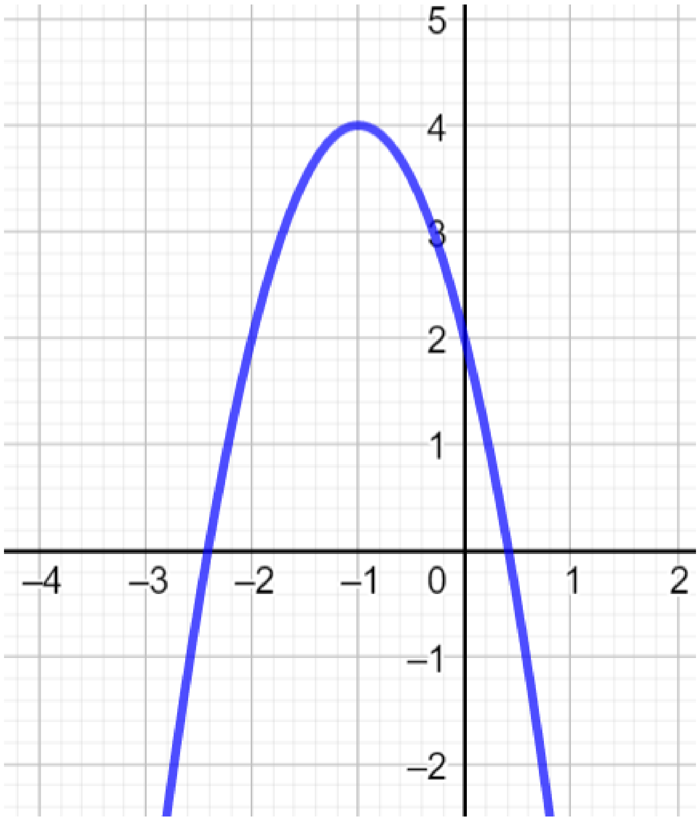
$-2x^2 - 4x + 2 = 0$

$a = -2, \;\; b = -4, \;\; c = 2$

From quadratic formula

Real roots: $x_1 = \frac{-2-\sqrt{8}}{2} \approx -2.4142,$

$x_2 = (-2+\sqrt{8})/2 \approx 0.4142$



Examples of Polynomial Functions (8/8):

$$f(x) = -2x^2 - 4x + 2$$

Real Roots:

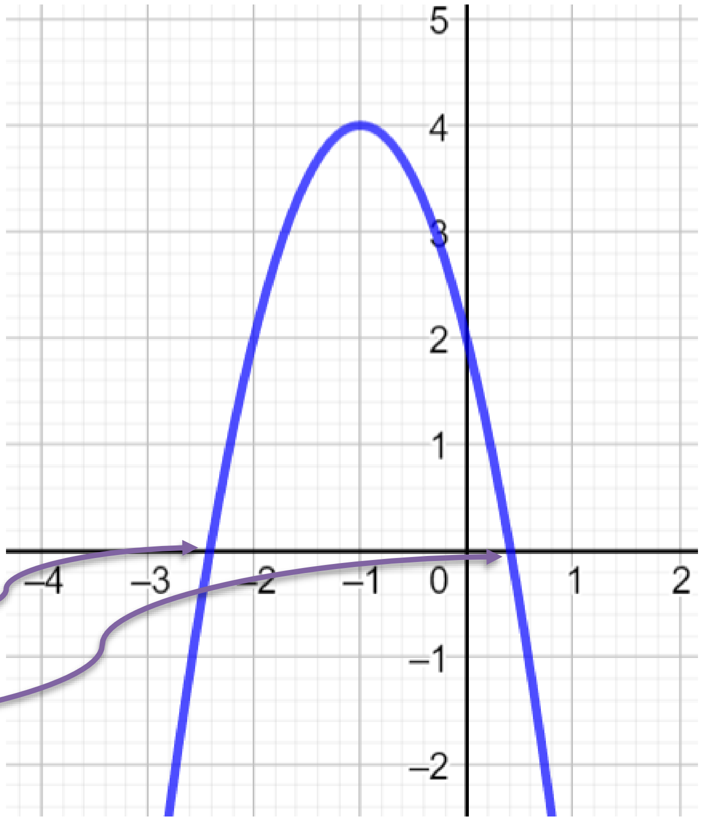
$-2x^2 - 4x + 2 = 0$

$a = -2, \quad b = -4, \quad c = 2$

From quadratic formula

Real roots: $x_1 = \frac{-2-\sqrt{8}}{2} \approx -2.4142,$

$x_2 = (-2 + \sqrt{8})/2 \approx 0.4142$

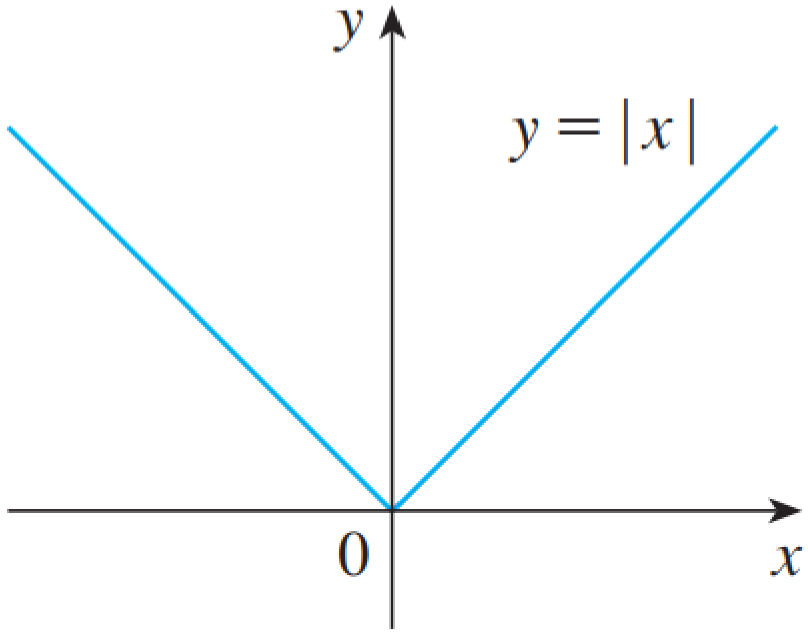


Types of Fun. & Graph (13/18)

Piecewise Defined Functions (1/4):

The absolute value function $f(x) = |x|$

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$$



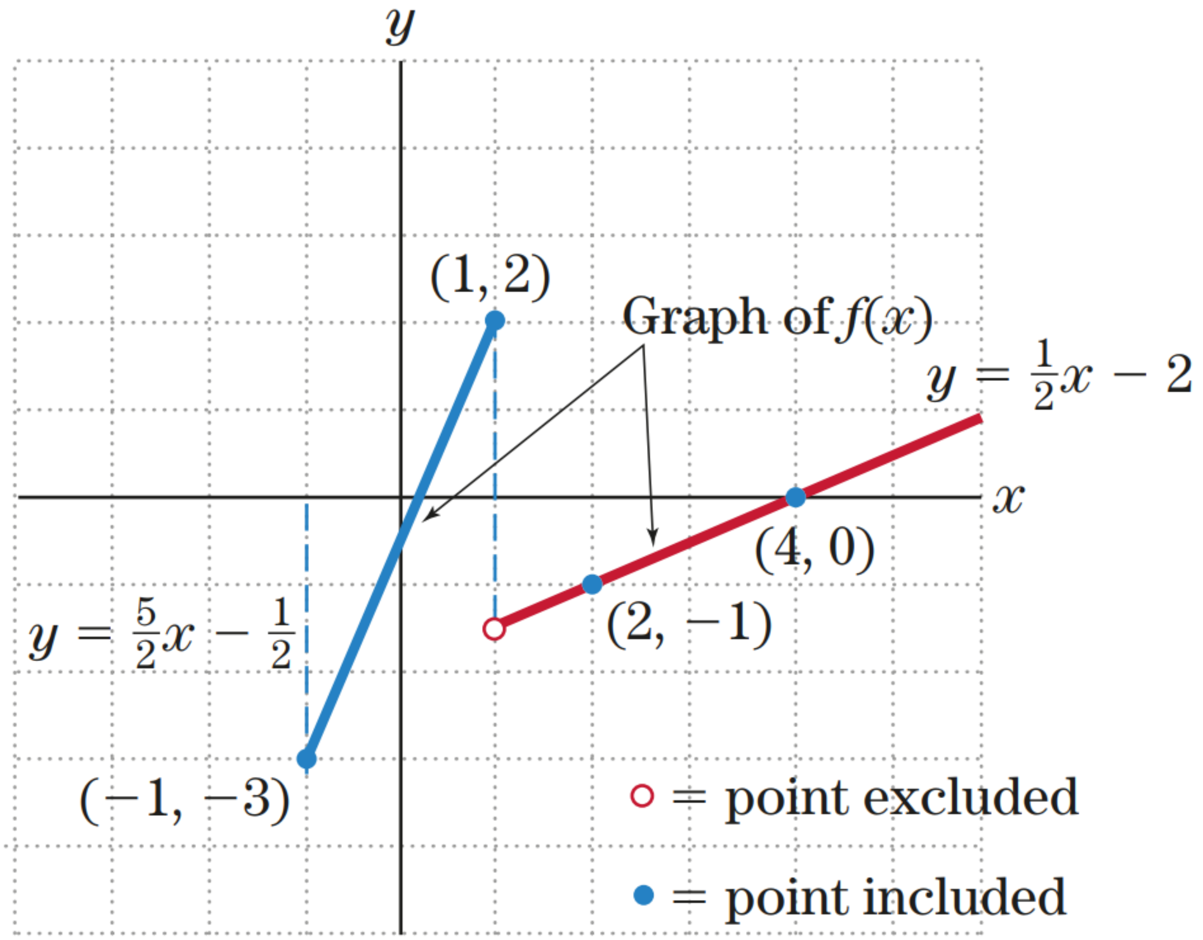
Types of Fun. & Graph (13/18)

Piecewise Defined Functions (2/4):

The piecewise-defined function

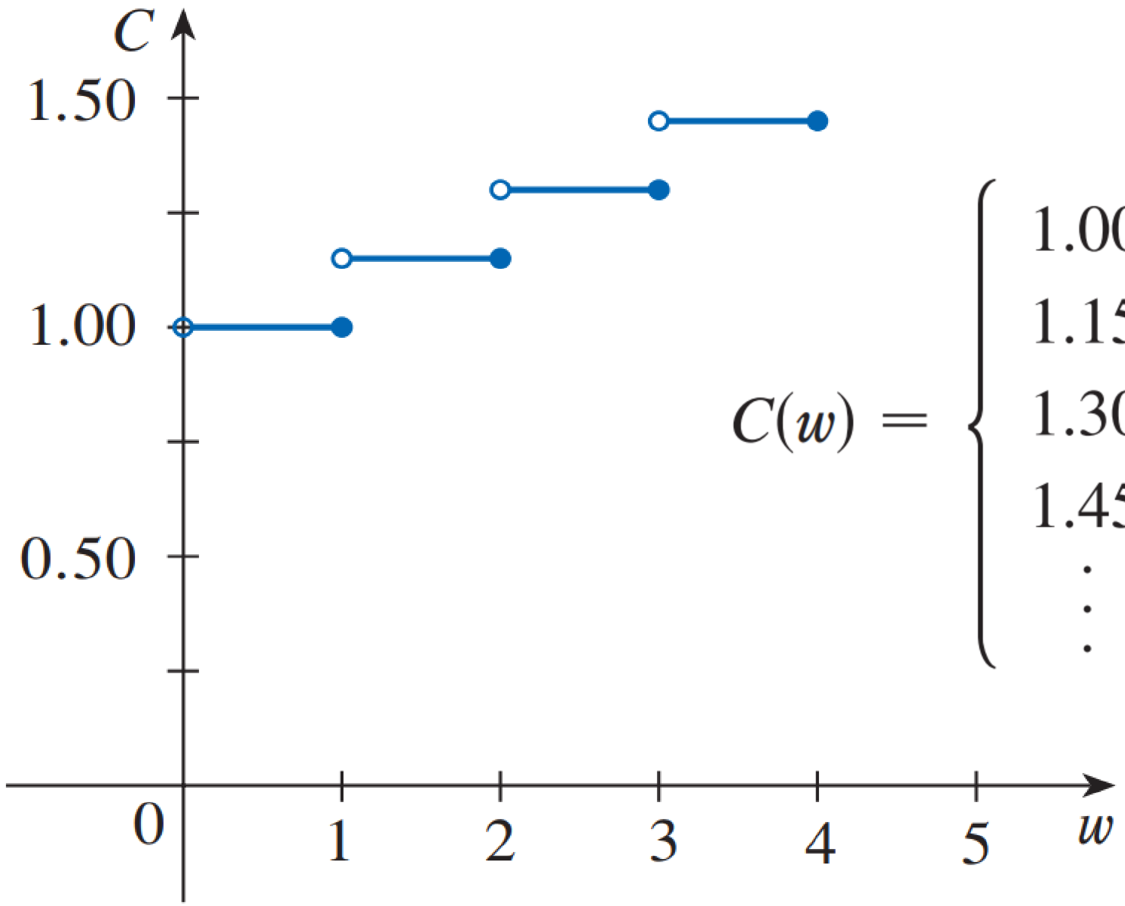
$$f(x) = \begin{cases} \frac{5}{2}x - \frac{1}{2} & \text{for } -1 \leq x \leq 1 \\ \frac{1}{2}x - 2 & \text{for } x > 1 \end{cases}$$

Piecewise Defined Functions (3/4):



Types of Fun. & Graph (13/18)

Piecewise Defined Functions (4/4):



$$C(w) = \begin{cases} 1.00 & \text{if } 0 < w \leqslant 1 \\ 1.15 & \text{if } 1 < w \leqslant 2 \\ 1.30 & \text{if } 2 < w \leqslant 3 \\ 1.45 & \text{if } 3 < w \leqslant 4 \\ \vdots & \end{cases}$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gkSI_PPAVJpebKDLo-ijEC

Lecture #2: https://www.youtube.com/watch?v=09wg9Pgpa6c&list=PLxIvc-MGOs6gkSI_PPAVJpebKDLo-ijEC&index=2 From 01:41:35

https://www.youtube.com/watch?v=_a2_RqTI0vs&list=PLxIvc-MGOs6gkSI_PPAVJpebKDLo-ijEC&index=3

Thank You

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